Characterizing financial crises through the spectrum of high frequency data.*

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Abstract

Financial market data in crises are usually modelled as possessing characteristics in common with non-crisis data and some additional peculiarities. Recent advances in analytical tools available for high frequency data make it possible to characterize which components of data generating processes change in crisis, and which do not. We introduce a set of new statistics which particularly indicate changes in tail behaviour across different sample periods. In an empirical application to US Treasury markets we find increased identification of price discontinuities and tail activity during the crisis period and evidence for flight to quality and cash across the term structure.

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1 Introduction

The behavior of asset returns between a non-crisis and crisis period differs in observable ways. Stylized facts for crisis period returns are that they are more volatile with large negative extremes, often negatively skewed and display increased kurtosis compared with non-crisis periods. These changes have been attributed to such things as idiosyncratic shocks, the transmission of shocks from other markets, and changing market conditions.[1] Crises are characterized as 'fast and furious' (see Kaminsky, Reinhart and Vega, 2003). Given this, it is surprising that to date there has been relatively little work on crises using high frequency data.

This paper fills that gap. It extends the recent framework of Aït-Sahalia and Jacod (2010a) on characterizing high frequency financial markets to formally test the proposition that the underlying data generating process for an asset remains the same across non-crisis and crisis periods [2]. In doing so the paper makes a number of contributions. First, it provides the first usage of high frequency spectrum statistics to compare financial markets across sample periods, and indirectly provides the first characterization of the US Treasury market using these tools. Second, it provokes a thorough evaluation of the proposed high frequency spectrum statistics, particularly with respect to variations in the chosen level of power for calculation, culminating in a series of experiments to extend the two dimensional representations of Aït-Sahalia and Jacod (2010a) to three dimensions. Third, it introduces a new statistic which specifically focuses on changing tail behavior between two sample periods, drawing on the findings that the most identifiable change in a crisis is contained in the tail observations, with potentially asymmetric tail changes being specifically acknowledged.

The Aït-Sahalia and Jacod framework uses a two dimensional graphical representation of what is actually a multidimensional statistic. In particular, results

[1] For example almost all existing models of the transmission of crises through contagion can be characterized this way. See the overview of modelling frameworks in Dungey et al (2005).

[2] Ait-Sahalia and Jacod (2010a) summarizes the research produced by these authors in Ait-Sahalia and Jacod (2010b,2009a,b,2008) and is related particularly to work by Tauchen and Todorov (2010a,b).

compiled with different power are commonly grouped. To overcome this we extend the representation to three dimensions and simulate a number of standard models to show how the statistics behave in both two and three dimensions, revealing further refinements about the data generating process. This three dimensional representation can separate two data sets which have seemingly identical properties in two dimensions.

The results from the high frequency data show that the features which distinguish crisis from non-crisis periods are contained in the tails. We propose new measures which extend the Aït-Sahalia and Jacod (2010a) tests to tail behavior only. We then construct a ratio statistic which provides an easily implementable test of the extent to which market behavior has changed between two sample periods.

To illustrate the use of high frequency statistics for crisis data we apply the statistics to secondary trading in US Treasuries on the BG Cantor platform during the financial crisis period from July 2007 until December 2008 compared with the pre-crisis period from July 2004 to July 2007. We find clear evidence that the 2,5,10 and 30 year bond returns retain Brownian motion and the presence of jumps across non-crisis and crisis periods. The crisis period is distinguished by an increased identification of jump activity. (Similar results can be found for US equity futures contracts and exchange rates, although we do not report these here.) Importantly, this result establishes that while price discontinuities are not necessarily more prevalent during crises, they are more easily identified. The new test provides strong evidence of the flight to cash effect - the mass of returns in the left tail for longer dated maturities increases during crisis, but for short dated maturities is dramatically reduced compared with the non-crisis period.

The paper proceeds as follows. Section 2 outlines the framework suggested for characterizing high frequency data drawing on the papers by Aït-Sahalia and Jacod (2010a, 2009a,b, 2008). It then shows problems which may arise in using the existing two dimensional representation of these statistics to compare non-crisis and crisis period data using simulation studies to produce a three dimensional representation. This information is then used to construct new statistics with which to make the comparison. Section 3 provides the application to US Treasury markets over the financial crisis period of 2007-2008 and shows statistically discernible changes between non-crisis and crisis periods, and in particular evidence of flight to the short-end of the market. Section 4 concludes.

2 Measures of high frequency market characteristics

Following Aït-Sahalia and Jacod (2010a) assume that the process describing prices for an individual asset, denoted X_t, is described by a semimartingale process of the form

$$\begin{aligned} X_t &= X_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW + \\ & \int_0^t \int_{[|x| \leq \varepsilon]} x(\mu - v)(ds_x, dx) + \int_0^t \int_{[|x| > \varepsilon]} x\mu(ds_x, dx) \end{aligned} \tag{1}$$

consisting of a sum of five potential components which are, in sequential order, a potentially non-zero mean, drift, a continuous component with Brownian motion and finally two jump terms, with μ the jump measure of X_t and v the Levy measure of its predictable component. The two jump terms represent firstly potentially many small jumps, no larger than some chosen threshold, ε, and a (finite) number of larger jumps which exceed the threshold size. The first step in characterizing the data is to decide which of the components given in equation (1) are useful in describing the particular data series of interest. A given series may be comprised of a pure jump process, not requiring the continuous component, or a purely continuous process with no jumps, or some mixture between these.[3]

[3] The activity signature function of Todorov and Tauchen (2010a) examines the same decomposition.

2.1 Characterizing high frequency returns

In practice we work with the discretized form of equation (1) and use equally spaced transaction data. ΔX_i^n refers to the change in the price in X between observation i and $i-1$ where the data are discretized by sampling at regular intervals given by the interval Δ_n, where an integer $k > 1$ can be used to denote lower sampling frequency as the interval $k\Delta_n$ Asymptotic results apply around $\Delta_n \to 0$, and there are T/Δ_n intervals in the given period of interest which contains a fixed T.

Consider two characteristics which can be used to construct descriptive statistics for the discretely sampled data series. These are:

(i) realized power without truncation

$$B(p, \infty, \Delta_n) = \sum_{i=1}^{T/\Delta_n} |\Delta_i^n X|^p \tag{2}$$

(ii) realized power with truncation

$$B(p, u_n, \Delta_n) = \sum_{i=1}^{T/\Delta_n} |\Delta_i^n X|^p \, 1_{\{|\Delta_i^n X| \leq u_n\}} \tag{3}$$

where u_n is the truncation level and the choice of $u_n = \alpha\Delta_n^{\varpi}$ where $\varpi \in (0, 1/2)$ and $\alpha > 0$ retains Brownian motion. The convenience of the measures given in equations (2) and (3) is that the behavior of a given series can be characterized by examining this set of statistics with changing power, p, truncation level, u_n and sampling frequency, k.

Aït-Sahalia and Jacod (2010a) provide a formal set of tests for; the presence of jumps, whether jumps represent finite activity and finally, whether the data is best represented as a pure jump process, that is there is no evidence of Brownian motion.[4] These tests are summarized below.

Proposition 1 *The statistic*

$$S_J(p, k, \Delta_n) = \frac{B(p, \infty, k\Delta_n)_T}{B(p, \infty, \Delta_n)_T}$$

[4] Aït-Sahalia and Jacod (2009a) also include tests for infinite activity and no Brownian motion, but as they are not germane to the application here they are ommitted from our discussion.

for $p > 2, k \geq 2$ and the total sample, converges in the limit to 1 for a process with jumps, and to $k^{p/2-1}$ for the no jumps case, and hence can be used for the presence of jumps.

The proof can be found in Aït-Sahalia and Jacod (2009b).

Proposition 2 *The statistic*

$$S_{FA}(p, u_n, k, \Delta_n) = \frac{B(p, u_n, k\Delta_n)}{B(p, u_n, \Delta_n)}$$

where $p > 2, k \geq 2$ and $u_n > 0$ converges in limit to $k^{p/2-1}$ in the case of finitely many jumps, and to the value 1 under the alternative of infinitely many jumps.

The corresponding central limit theorem can be found in Aït-Sahalia and Jacod (2008).

Proposition 3 *The statistic*

$$S_W(p, u_n, k, \Delta_n) = \frac{B(p, u_n, \Delta_n)}{B(p, u_n, k\Delta_n)}$$

where $p < 2$ and $k \geq 2$ and an integer, converges to $k^{1-p/2}$ for a process with Brownian motion, and alternatively to 1.

The proof may be found in Aït-Sahalia and Jacod (2010b).

Note that the statistic S_W is the simple inverse of S_{FA} but is being assessed over a different range of p, which is where the direct correspondence with the graphical activity signature function of Todorov and Tauchen (2010a) arises.

The measures given above are for the case of no microstructure noise. In the case where there is potentially additive market microstructure noise, S_J and S_{FA} converge to $1/k$ where noise dominates and $1/k^{1/2}$ when rounding errors dominate. S_W converges to k when noise dominates and $k^{1/2}$ when rounding error dominates.

The statistics above cover the case of the whole distribution, S_J, and the truncated distribution, S_{FA} and S_W. To fill the missing section of the distribution between these tests we establish a new statistic to examine the behavior of the tails of the distribution alone, which we denote a test of tail intensity as follows:

Proposition 4 *The statistic*

$$S_{TI}\left(p,u_n,k,\Delta_n\right)=\frac{\sum_{i=1}^{T/\Delta_n}\left|k\Delta_i^n X\right|^p 1_{\left\{\left|\Delta_i^n X\right|>u_n\right\}}}{\sum_{i=1}^{T/\Delta_n}\left|\Delta_i^n X\right|^p 1_{\left\{\left|\Delta_i^n X\right|>u_n\right\}}}.$$

converges to 1 *in the case of large jumps and* $k^{p/2-1}$ *where the series has no large jumps. In the presence of additive microstructure noise* S_{TI} *will converge to* $1/k$ *when noise dominates and* $1/k^{1/2}$ *when rounding error dominates.*

The proofs of these properties are trivial as they are precisely analogous to those for S_J and S_{FA} presented in Aït-Sahalia and Jacod (2010b, 2009b 2008) and are hence omitted for brevity. An advantage of examining the tails themselves is that potential asymmetry can be accommodated, resulting in two further statistics for positive and negative tail intensity as

Proposition 5 *The statistic*

$$S_{TI}^{+}\left(p,u_n,k,\Delta_n\right)=\frac{\sum_{i=1}^{T/\Delta_n}\left|k\Delta_i^n X\right|^p_{\left\{k\Delta_i^n X>u_n\right\}}}{\sum_{i=1}^{T/\Delta_n}\left|\Delta_i^n X\right|^p_{\left\{\Delta_i^n X>u_n\right\}}}$$

converges to 1 *in the case of large positive jumps and* $k^{p/2-1}$ *in the absence of large positive jumps.*

Proposition 6 *The statistic*

$$S_{TI}^{-}\left(p,u_n,k,\Delta_n\right)=\frac{\sum_{i=1}^{T/\Delta_n}\left|k\Delta_i^n X\right|^p_{\left\{k\Delta_i^n X<-u_n\right\}}}{\sum_{i=1}^{T/\Delta_n}\left|\Delta_i^n X\right|^p_{\left\{\Delta_i^n X<-u_n\right\}}}$$

converges to 1 *in the case of large negative jumps and* $k^{p/2-1}$ *in the absence of large negative jumps.*

2.2 Distinguishing Crisis Periods

This paper particularly considers how the descriptive statistics for high frequency financial returns described above may differ between non-crisis and crisis periods. In this way we are able to pin down the important components of behavior in financial returns, which then allows us to link more clearly with the literature on crisis transmission. In particular, if the characterization of jumps or tail behavior change then this links strongly to the literature on non-linear transmission mechanisms which single out large or atypical observations as the key to understanding crises, as for example in Bae, Karolyi and Stulz (2003), Boyson, Staehl and Stulz (2010), while if the Brownian motion or finite activity properties of the data change this supports alterations of the fundamental process, such as the latent factor model approach of Billio and Caporin (2010) or copula based approach of Busetti and Harvey (2011) for example.

We approach this problem in two ways. The first is to compare the distribution of statistics suggested by Aït-Sahalia and Jacod (2010a) as generated by Tests 1 to 3 for non-crisis and crisis periods and formally test whether they are significantly different. To generate these distributions each statistic is constructed for subsamples at a variety of power and sampling frequencies. Concatenating the generated statistics by power and sampling frequency forms a histogram of the test statistic for the asset return in question. This is then used to determine whether the statistic supports the null hypothesis or otherwise. A direct comparison of the distributions for two sample periods can be tested using a Kolmogorov-Smirnov test.

However, concatenating the test statistics by power and sampling frequency may not desirable. In some cases the resulting test statistics will produce outcomes which are rather difficult to interpret. More fundamentally, the representation may mislead the researcher. Consider, for example, the case for S_J when $p = 2.1$ and $k = 2$. The value associated with H_0 is 1 and the value associated with H_A under the no microstructure noise case is $k^{0.1}$ giving the outcomes for $k = \{2, 3\}$ of $\{1.07, 1.12\}$. If, however, $p = 4$, H_0 is 1 and the value associated with H_A under no microstructure noise is k^1 giving outcomes of 2 and 3 for $k = \{2, 3\}$ respectively. This means that rejections of H_0 at low values of p contribute to mass around 1 which tend to make the histograms appear to support H_0 at all values of p.

As will be shown in the empirical example this problem is most apparent in the S_J statistic. In the next section simulations address the extent of these problems using the S_J statistic as the exemplar. In particular we examine three dimensional representations of S_J, p, and frequency, while controlling k to a single value. In this way we avoid mixing the results in a counter-intuitive manner.

2.3 Simulation Results

The two dimensional representation and a three dimensional representation of the test statistic S_J are generated for a number of models. As baseline processes we consider three cases: Brownian motion, a regime switching Brownian motion with 10 potential regimes and 5 orders of magnitude between the volatility of the least and most volatile regimes with random switching, and a skewed distribution based on Azzalini (1995).[5] Each of these baselines were simulated for the cases of no jumps, small jumps and large jumps.[6]

The simulation parameters are calibrated to a liquid equity, the same as those used in Aït-Sahalia and Jacod (2009a), which in turn are loosely based on the estimates of the Heston model for S&P500 data in Aït-Sahalia and Kimmel (2007). The sampling frequency used in their calibration (approximately every 5 seconds) is far in excess of the liquidity of the data samples in the applications

[5] A continuous random variable Z has a probability density function $f(z) = 2\phi(z)\Phi(\alpha z)$ where α is a shape parameter influencing the density function $\Phi(\alpha z) = \int_{-\infty}^{\alpha z} \phi(t)$ and $\phi(z) = \exp\left(\frac{-z^2}{2}\right)/\sqrt{2\pi}$. In the simulations shape parameter $\alpha = 4$, implying a skewness of 0.78 consistent with that observed in financial returns data; see for example Fry, Martin and Tang, 2010.

[6] We conducted similar experiments for a Heston process which produced results corresponding to those for the Brownian motion, and for Brownian motion with skewness and regimes which produced results corresponding to those for the regimes simulation. We also simulated the case of Brownian motion with rounding for which results looked similar to Btrownian motion with small or large jumps. These results are available on request.

of this paper, so our calibrations scaled to 201 observations per day.[7] For all processes we simulate 6000 days of observations, producing 60,300 seconds of observations per day from which we extract 201 data points corresponding to 5 minute observations. This gives the equivalent of 100 quarters of observations based on 20 days per month. Across that dataset we randomly assign 10,000 potential jumps, where small jumps are implemented by an additional random element drawn from the normal distribution and scaled by a factor of 3. Large jumps are implemented by multiplying the random element by 6. The initial price level in the process is set to 1000.

Figure 1 gives the two dimensional representation of the simulated statistics for each experiment with the proportion of observations on the vertical axis and the value of the S_J statistic on the horizontal axis as suggested in Aït-Sahalia and Jacod (2010a). Figure 2 gives the corresponding three dimensional representation with the frequency of occurrence of the value of S_J grouped by the power, p, where there were 1000 realizations of each value of p.

In the cases of Brownian motion with no jumps the theoretical result that the mass is centred on the value $k^{p/2-1}$ (where $k = 2$) is clearly evident in the three dimensional results where the range of hills can be viewed as curving away with higher p. It is also apparent that as p increases, the concentration of distribution becomes more diffuse, although the greatest mass remains at $k^{p/2-1}$. It is evident by comparing the corresponding chart in Figure 1, that while the shape of the Brownian motion function is retained, the larger p clearly result in increasing values of S_J, due to increased variance in the measure. The choice of p at which to evaluate the statistic will make a difference to how well we are able to identify different outcomes from two dimensional figures.

The addition of small jumps to the Brownian motion show that this results in a somewhat steeper drop off in the distribution of the mass of the statistic than in the simple Brownian motion case, while in the case of large jumps the mass is centred clearly on 1 at all values of p examined, although this again becomes

[7] The calibration to 201 5 minute observations per day represents the trading frequency of overnight equity futures indices for example.

more diffuse as p increases. The difficulties of visually distinguish between the Brownian motion case and that of Brownian motion with jumps are clearly evident, particularly in the two dimensional representation.

In the switching model simulation the case with no jumps looks very similar to the simple Brownian motion shown immediately above it. However, when small or large jumps are introduced the distribution of the statistic is strongly centred on 1 meaning that jumps are strongly detected even in the small jumps case. The three dimensional figure does not contain the same extent of widening in variance with increased p as in the benchmark Brownian motion case.

Skewed returns without jumps have a slightly differently distributed mass to the previous cases, although this is difficult to detect in Figure 1. The addition of small jumps in a skewed environment seems to increase the difficulty in deciding whether jumps are dominant. The median in the two dimensional figure is clearly above 1, but there is evidence of increasing variance around an increasing mean at each value of p in the three dimensional representation.

Figure 3 presents the two dimensional statistics for each simulation on a single figure. It shows that visually there is little difference in the no jumps representations between the experiments, although the skewed data will produce more mass closer to $S_J = 1$, an important consideration given that increased skewness is commonly associated with crisis periods. In the case of small jumps, the differing outcomes are readily apparent. Regime switching tends to emphasize the presence of jumps, possibly as regime shifts are being incorporated as jumps. Skewness also increases the mass of the distribution closer to 1. In the case of large jumps, Figure 3 shows that all distributions find a clustering around 1, although this is less pronounced in the case of simple Brownian motion with large jumps. This evidence supports the contention that other features of the data may well be biasing our visual categorization of the process towards accepting the presence of jumps in a given series, particularly in the presence of either skewness or changes in regime.

2.4 Crisis detection statistics

We wish to compare the behavior of the different statistics in non-crisis and crisis periods. In particular, it turns out that we are interested in the behavior of the tails, and particularly the S_{TI}, S_{TI}^{+} and S_{TI}^{-} statistics across the two samples.[8] We introduce the S statistic, constructed as the ratio of S_{TI} statistics in the crisis period compared with the non-crisis period. It has three forms, one for the combined tail behavior S and one for each of the positive and negative tails, S^{+}, S^{-}, as follows:

Proposition 7 *The statistic*

$$S\left(p, u_n, k, \Delta_n\right) = \frac{S_{TI,crisis}\left(p, u_n, k, \Delta_n\right)}{S_{TI,noncrisis}\left(p, u_n, k, \Delta_n\right)} \tag{4}$$

converges to 1 *for all* p *where the tail behavior of the returns is the same in both sample periods.*

Proposition 8 *The statistic*

$$S^{+}\left(p, u_n, k, \Delta_n\right) = \frac{S_{TI,crisis}^{+}\left(p, u_n, k, \Delta_n\right)}{S_{TI,noncrisis}^{+}\left(p, u_n, k, \Delta_n\right)} \tag{5}$$

converges to 1 *for all* p *where the positive tail behavior of the returns is the same in both sample periods.*

Proposition 9 *The statistic*

$$S^{-}\left(p, u_n, k, \Delta_n\right) = \frac{S_{TI,crisis}^{-}\left(p, u_n, k, \Delta_n\right)}{S_{TI,noncrisis}^{-}\left(p, u_n, k, \Delta_n\right)} \tag{6}$$

converges to 1 *for all* p *where the negative tail behavior of the returns is the same in both sample periods.*

If there is no change in the tail distributions, then each of the statistics will have the value 1, while an increase in the mass in the tails in the crisis period will be indicated by $S > 1$, and a decrease by $S < 1$. An increase in the mass in the right hand tail (left hand tail) will be indicated by $S^{+} > 1$ ($S^{-} > 1$), and a decrease by $S^{+} < 1$, ($S^{-} < 1$).

[8] Clearly we could also construct analagous statistics to compare S_J, S_{FA} and S_W across the two samples. However, to pre-empt our empirical results, interest rests solely with the tail indicators.

3 Empirical Example: The US Treasury market

The secondary market for US Treasuries is one of the largest individual asset markets in the world with turnover of over $US120 trillion in 2008. The majority of trade since the turn of the century has migrated to two dominant electronic communication networks (ECNs), one now known as BGCantor (formerly eSpeed) and the other BrokerTec. The existing empirical evidence suggests that total turnover is reasonably evenly divided between them - Mizrach and Neely (2006) find approximately 40% of ECN turnover on BGCantor, but more recent comparisons in Jiang et al (2010) and Dungey, McKenzie and Smith (2009) make it closer to 50% each. The US Treasury market played a key role in the flight to liquidity and quality which occurred in the crisis of 2007-2008 as it did during for the period of the Hong Kong dollar speculative double play and the near collapse of the US based hedge fund Long-Term Capital Markets in August and September 1998 in Dungey, Fakhrutdinova and Goodhart (2009).

In order to examine the differences between non-crisis and crisis period trading, we divide our total sample period of July 1, 2004 to December 31, 2008 into a calm period beginning with the upswing of interest rates from the previous US monetary policy cycle, and a crisis period beginning July 17, 2007. The breakpoint between the pre-crisis and crisis data is consistent with the first indications of troubles from hedge funds at Bear-Stearns and precedes the changes in policy at the European Central Bank on August 9, 2009. Both of these dates have been used to mark the start of the crisis elsewhere.[9] The data cover the main US trading hours of 8:00am to 5:30pm EST for the secondary trading in 2, 5, 10 and 30 year bonds.

Figure 4 presents the realized variance from 5 minute returns in each maturity across the sample period, the average daily realized variances for the two subperiods are given in Table 1. There is a clear increase in variance (and covariance which is not reported) between assets during the crisis period compared with the relatively tranquil non-crisis period. Large increases in realized

[9] There are by now, many chronologies of the 2007-2008 crisis, including Rose and Speigel (2009) and Bordo (2008).

variance occur in July-August 2007 around the beginning of the crisis, with greater rises in September-December 2008 during the period of the Lehman Bros. bankruptcy, the rescue of AIG, collapse of Fannie Mae and Freddie Mac and numerous other instances of institutional stress.

Unlike the applications reported in Aït-Sahalia and Jacod (2010a) sampling US Treasury transaction data in sub-minute intervals is not appropriate, as there would simply be too many intervals with no transaction. Evidence in Dungey, McKenzie and Smith (2009) and Dungey and Hvozdyk (2010) suggests that 5 minutes is an appropriate sampling frequency. We therefore proceed with the interval $\Delta_i^n = 5$ minutes.[10]

3.1 The pre-crisis and crisis statistics

3.1.1 Evidence for jumps: S_J

The empirical distributions of the test statistic for the presence of jumps S_J for the pre-crisis sample (solid line) and the crisis sample (dashed line) are shown in the first panel of Figure 5. Each line represents the values obtained for S_J as p varies across $2.1 \leq p \leq 6.0$ in 0.1 increments and $k = 2, 3$ pooled for all maturities as suggested in Aït-Sahalia and Jacod (2010a). It is clear that the median is just larger than 1 in each case, supporting evidence for jump activity. Comparing the two subsample distributions, it is apparent that the mass of the distribution has shifted to become right skewed during the crisis rather than left skewed in the pre-crisis period. A Kolmogorov-Smirnov test clearly rejects the null that the distributions are the same as reported in Table 2. The implication of this shift is that in the pre-crisis period jumps were less distinguishable from noise (represented by values of $S_J < 1$), whereas in the crisis period, almost all of the distribution supports the null of jump activity. The long right tail is more difficult to interpret, as right tails are more appropriately associated with Brownian motion. This motivates us to examine these figures in three

[10] The choice of optimal sampling regime for different assets is a topic of ongoing research, but efforts include Russell and Bandi (2006), Aït-Sahalia, Mykland and Zhang (2005). The consequences of incorrect sampling are either estimates corrupted by noise from over frequent sampling (and biased covariance estimators see Sheppard, 2006) or the exclusion of information from insufficently frequent sampling.

dimensions, similar to the presentation of the simulation results in Section 2.3.

The three dimensional representations of the S_J statistics for the Treasury data during the non-crisis and crisis periods are presented in the right hand panels of Figure 5. Two aspects of the representations are immediately apparent. The first is that the three dimensional representations are not strikingly similar to any of the simulated series presented in Section 2.3, despite the seeming correspondence between the two dimensional representations of the simulated data and the application data. The three dimensional representations have greater mass near the origin - that is for small values of p there is less variance around the mean S_J value than in the simulations. However, as p increases the mass quickly disperses, leaving greater variance at higher p than indicated in the simulations. The differences between the pre-crisis and crisis distributions exacerbates this tendency with even greater clustering of the mass around the mean for low values of p, and more dispersion at higher values. The practical implication of this finding is that crisis data can be distinguished from pre-crisis data by greater evidence of jump type behavior at lower power values. As p increases, there is increased uncertainty about comparisons across the two periods. The suggested value of $p = 4$ in Aït-Sahalia and Jacod (2009a) is unlikely to yield useful results here as the actual data has quite a flat distribution in both cases by this point. Lower values of p are likely to lead to more useful distinctions between the distributions. In essence, usefully implementing this tool requires an appropriate metric for choosing p. This strongly motivates the use of the S statistic developed in Section 2.4.

3.1.2 Evidence for Finite Activity: S_{FA}

Prior to turning to the S statistic, we first examine whether the jump activity detected in the previous subsection is finite using Test 2. To do so requires specification of u_n, chosen here to be the 1% tail of the absolute values of the 5 minute interval log returns in the pre-crisis period. The distribution of the calculated test statistic for whether jump activity is finite, S_{FA}, is given in the first panel of Figure 6, where the statistics are calculated across all maturities for $2.1 \leq p \leq 6.0$ in 0.1 increments and $k = 2, 3$. The mass of the distribution covers the range from 0.5 to 1.5 but both the pre-crisis and crisis distributions are centred on 1, consistent with infinite activity jumps. The distributions appear very similar in the two periods and the Kolmogorov-Smirnov test statistics reported in Table 2 report that the two distributions are insignificantly different.

3.1.3 Evidence for Brownian Motion: S_W

The second panel of Figure 6 plots the distribution of the S_W statistics calculated for each subsample for $1 \leq p \leq 1.75$ incrementing by 0.05, and for $k = 2, 3$. The striking aspect of the figure is that the results clearly support the presence of Brownian motion in the data generating process, and the distribution is hardly changed across the pre-crisis and crisis samples. This is confirmed by the Kolmogorov-Smirnov test statistics reported in Table 2.

3.1.4 Evidence on Tail Intensity: S_{TI}^{+} and S_{TI}^{-}

The right hand panels of Figure 6 contain the distribution of the S_{TI}^{+} and S_{TI}^{-} statistics. It is clear that the figures look quite different in the two samples. This is confirmed by the Kolmogorov-Smirnov test statistics reported in Table 2, and this is despite the fact that we have not at this stage separated the different maturities of bonds.

Comparing the results for these tests in the pre-crisis and crisis data samples, reassuringly confirms the stability of the form of the underlying process (as containing a continuous component and jumps), but emphasizes that there is greater certainty in distinguishing the presence of jumps from mere noise during a crisis, revealing the increased presence of large jumps during the crisis period. In particular the evidence suggests changes to the S_{TI}^{+} and S_{TI}^{-} distributions. To examine this further we now consider formally testing whether the tail behavior has changed between the non-crisis and crisis period using the S, S^{+} and S^{-} statistics by individual maturity from Section 2.4.

3.2 Changing behavior in the crisis

Applying the S statistics across incrementing values of p to the US Treasury data gives the profiles shown in Figure 7 for sampling frequencies $k = 2, 3$ in the top and bottom panels of the Figure. The left hand panels of Figure 7 reveal that S is greater than 1 for all values of p, for both $k = 2, 3$, indicating an increasing mass in the tails during the crisis period - consistent with increased volatility and fat tails during periods of stress. As expected from the three dimensional results, the outcome varies with p, in this case S is generally monotonically increasing in p, and this is particularly evident for the longer maturities. However, this is not uniformly the case. For $k = 3$ (which compares 15 and 5 minute sampling) the 5 year bonds show a greater deviation between the non-crisis and crisis samples at mid range powers, around $p = 3.3$.

Formal tests of the slope of each of the lines plotted in Figure 7 are reported in Table 5,which are constructed as a test of the hypothesis that $S = 1$ for all p. The results show that the null hypothesis of a horizontal line is rejected for all maturities and at both sampling frequencies ($k = 2, 3$). At this point it is worth noting that we place more value on the $k = 2$ case, comparing 5 and 10 minutes sampling as 15 minute sampling from the $k = 3$ case captures relatively few observations for a single trading day.

The S^+ statistics shown in the central panels of Figure 7, and the corresponding tests $S^+ = 1$ for all p in Table 3, also strongly support greater mass in the right hand tail during the crisis period, consistent with existing results about increasing right hand skewness during crisis periods. All the tests report increased mass in the positive tails.

The final case presented, of S^-, provides an interesting contrast. In this case, the long maturities portray definite increases in mass in the negative tails, with this being more apparent for lower powers in the 30 year maturities (see the right hand panels of Figure 7). In the case of the short maturities, however, the mass in the negative tail is reduced. The formal tests of whether $S^- = 1$ for all p reported in Table 3 accept the null for the longest maturities at both

$k = 2$ and $k = 3$. These longest maturities show increased mass at the negative tail, consistent with the findings for the positive tail, indicating that there are more tail (extreme) observations during the crisis in both negative and positive tails. However, at the short end the null is rejected for the 2 year maturities for $k = 2$, indicating that there is decreased mass in the tail. For the 2 and 5 year maturities in the $k = 3$ case, there is insufficient evidence to reject the null of no change in the mass in the negative tail.

While the short end of S^+recorded increased mass, the negative tail displays strong evidence of decreased or unchanged mass. This is clear evidence of flight to cash in the data. While negative returns became more prevalent for longer maturities, they did not for short dated Treasuries. The flight away from maturity, towards quality and cash is a characteristic of the credit crunch conditions of the crisis period. This result is strong evidence of the potential usefulness of this result in moving towards a crisis detection mechanism based on high frequency data.

The conclusion from the empirical results presented here is that the underlying form of the data generating process for individual series is not disrupted by the advent of a financial crisis. That is the series continue to display characteristics consistent with a continuous component with jumps. What changes during financial crises is the nature of the jumps process. The results show that it becomes easier to distinguish jumps from noise during a crisis period, and that there are discernible shifts in the mass of the descriptive statistics which can be described as in the tail mass. Comparisons of the ratio of the tail mass of the S_{TI}, S_{TI}^{+}, and S_{TI}^{-} statistics calculated at different powers, p, in crisis and non-crisis periods provide indicators of the crisis events, and have results with economic interpretation, such as flight to quality and cash. Ongoing work considers whether these indicators can be used to endogenously date the occurrence of crises, particularly in a multiple asset world.

4 Conclusions

The behavior of financial market data during periods of financial stress is of great practical importance to investors, analysts and policy makers alike. Crisis models often assume that some underlying data generating process remains stable across both non-crisis and crisis periods, but is augmented by new or enhanced features during the crisis period. This paper develops a suite of tools to consider which aspects of data generating processes remain stable, and which change during a crisis. Applying these to data on US Treasury bonds data reveals that the evidence for Brownian motion and finite or infinite activity jumps is not significantly changed between a pre-crisis period and the financial crisis of 2007-2008. However, the statistics concerning the presence of jumps differ. The difference between the two periods supports a greater ability to discern jump activity from noise during the crisis period, with the distribution of the jump statistic more concentrated around the critical value associated with jumps and also displaying right skewness.

The results in the paper strongly support the contention that changes in the spectrum of financial market returns between non-crisis and crisis periods are evident in tail behavior. Further, the detection of these changes is affected by the choice of power in calculating summary statistics. Thus, we propose a new statistic based on the behavior of the tails only, treating left and right tails separately, to account for potentially changing skewness patterns identified in Fry, Martin and Tang (2010). This new statistic, specifically compares the truncated tails of the returns across two subsamples, and shows how differential masses in the tail result in deviations of the statistic from 1. In application to the US Treasury market data the results show increasing mass in right tails across maturities, but decreasing mass in the left tail of the short end of the maturity structure is clearly consistent with flight to quality and cash increasing the desirability of these assets.

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Table 1:
Descriptive Statistics in the pre-crisis period of July 1, 2004 to July 16, 2007 and crisis period of July 17, 2007 to December 31, 2008.

	Realized Variance							
	Pre-Crisis Period				Crisis Period			
	Mean	Std. Dev.	Min.	Max	Mean	Std. Dev.	Min.	Max.
2 year	0.006	0.006	0.001	0.074	0.033	0.036	0.003	0.339
5 year	0.034	0.038	0.004	0.429	0.154	0.148	0.021	1.242
10 year	0.095	0.091	0.012	1.106	0.322	0.267	0.046	1.809
30 year	0.295	0.247	0.043	2.774	1.001	0.863	0.150	6.558

Table 2:
Kolmogorov-Smirnov test statistics for equality between the pre-crisis and crisis cumulative density functions of the S_j, S_{FA},S_W, S_{TI} (p-values in parentheses).

	S_j	S_{FA}	S_W	S_{TI}^{+}	S_{TI}^{-}
Test statistic	8.380	1.186	0.938	9.881	2.649
	(0.000)	(0.120)	(0.343)	(0.000)	(0.000)

Table 3:
Test statistics of null hypothesis that S, S^{+} and S^{-} are horizontal lines, implying no change between the non-crisis and crisis period tail intensities (p-values in parentheses).

	S		S^{+}		S^{-}	
	$k = 2$	$k = 3$	$k = 2$	$k = 3$	$k = 2$	$k = 3$
2 year	1.408	6.169	-0.561	3.271	-3.152	-2.332
	(0.084)	(0.000)	(0.289)	(0.001)	(0.002)	(0.012)
5 year	1.723	0.982	-0.531	2.371	-4.689	-1.394
	(0.046)	(0.166	(0.299)	(0.011)	(0.000)	(0.086)
10 year	2.069	5.801	4.257	1.810	1.257	-0.569
	(0.023)	(0.000)	(0.000)	(0.039)	(0.108)	(0.287)
30 year	2.048	5.026	4.824	5.235	0.669	3.270
	(0.024)	(0.000)	(0.000)	(0.000)	(0.254)	(0.000)

Figure 1: Simulation Experiments I-IX, 2D representations of S_J with no jumps, small jumps and large jumps.

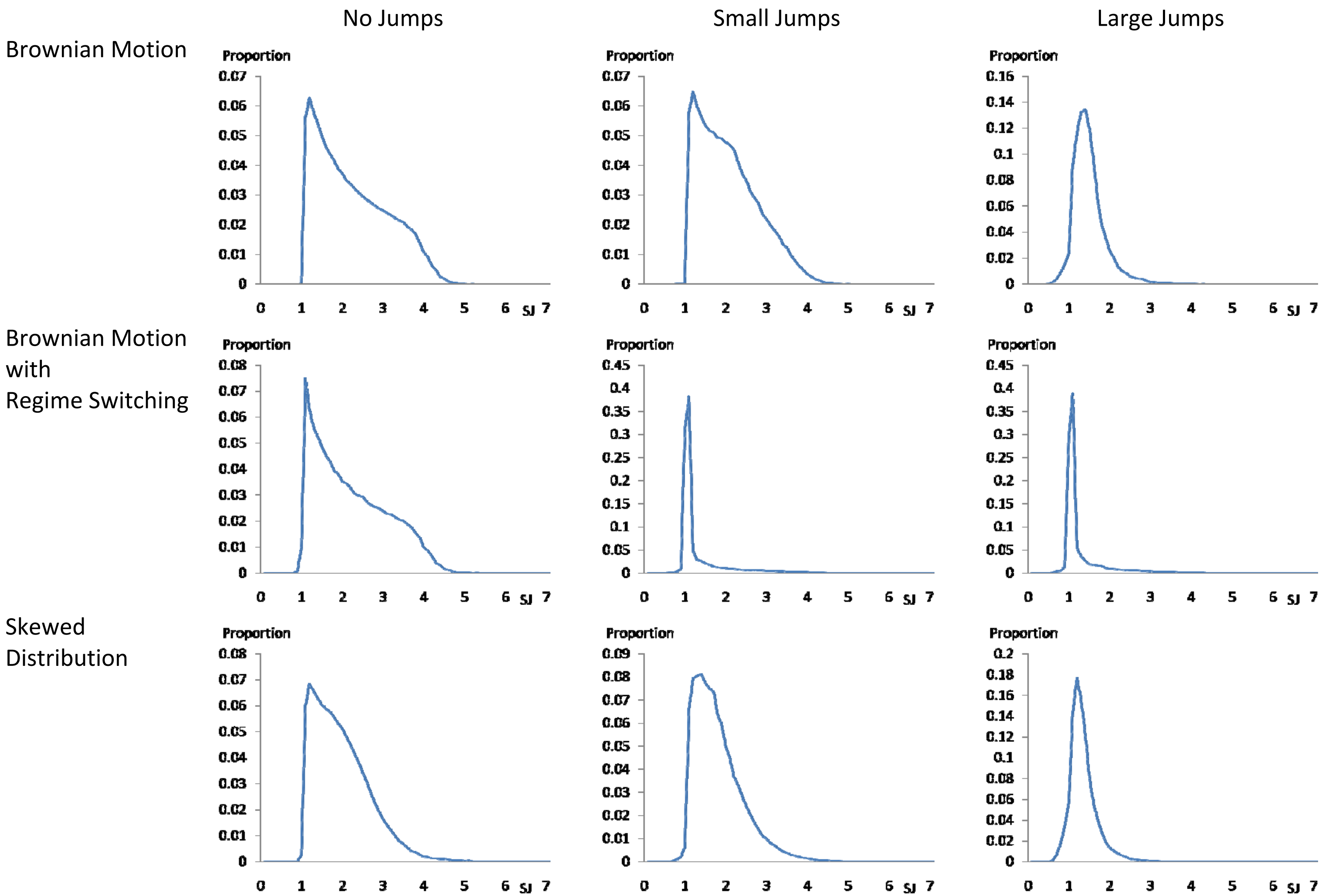


Figure 2: Simulation Experiments I-IX, 3D representations of S_J with no jumps, small jumps and large jumps.

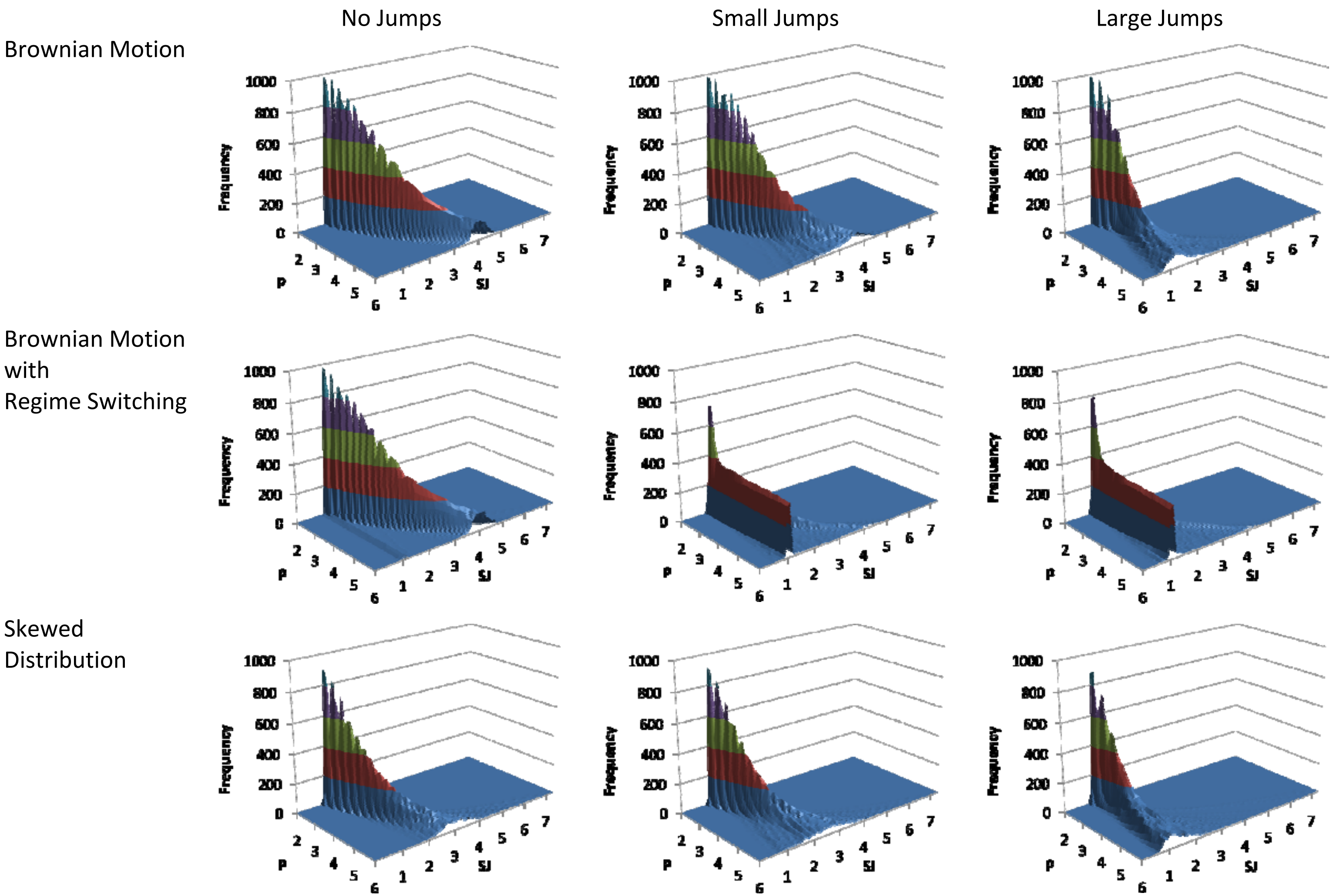


Figure 3 Comparison of 2D representations of the S_J statistics in various experiments

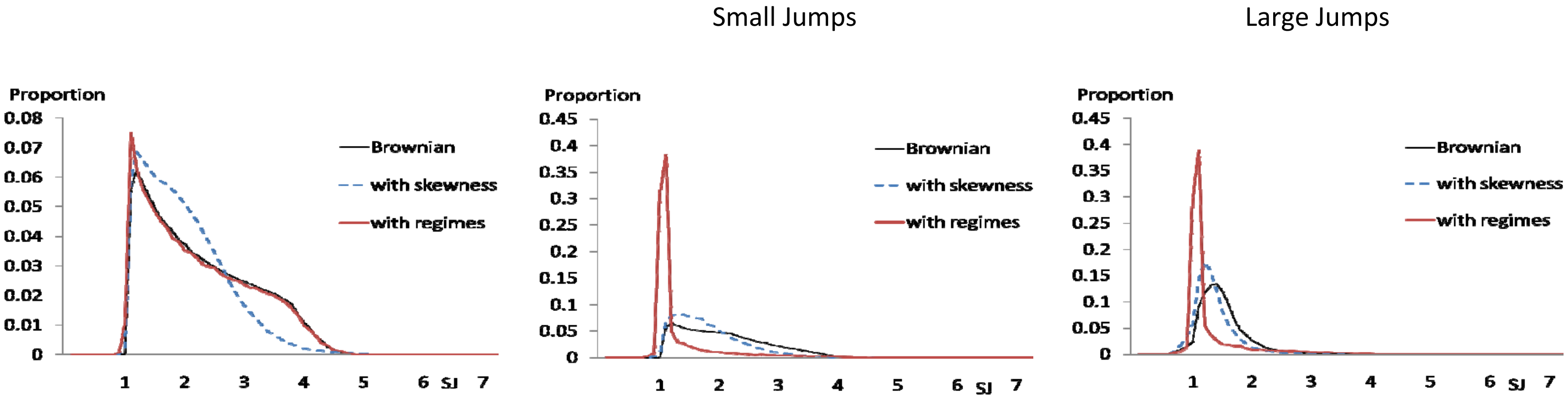


Figure 4: Realized Variance of 2,5,10 and 30 year maturity US Treasuries

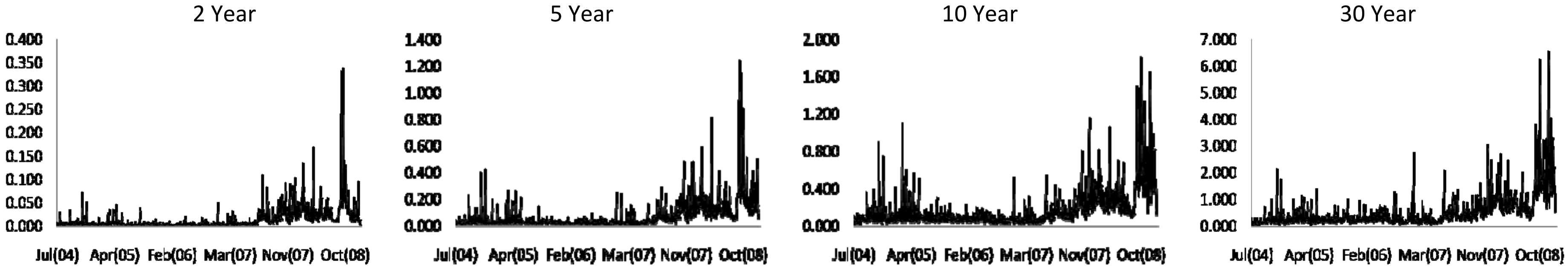
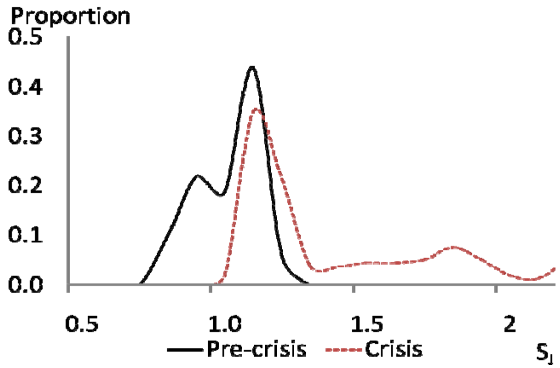
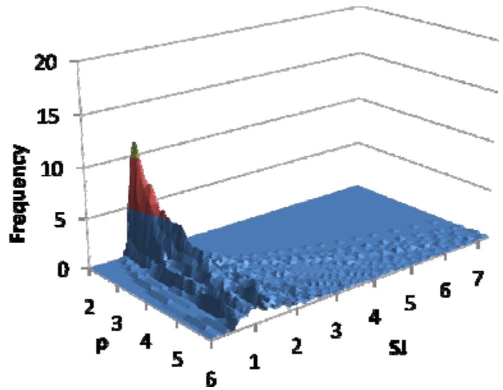


Figure 5 Representations of the S_J descriptive statistics for US Treasuries in non-crisis and crisis periods

2D representation non-crisis and crisis
2.1≤p≤6, k=2,3



3D representation non-crisis
2.1≤p≤6



3D representation crisis
1≤p≤1.75, k=2,3

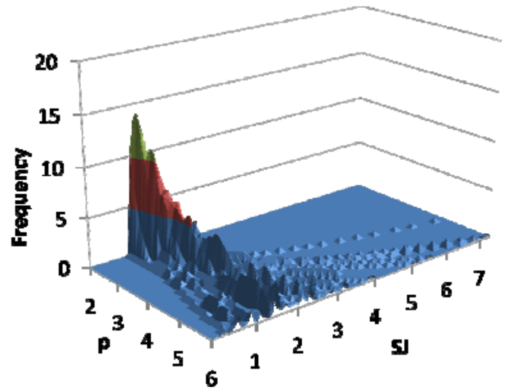
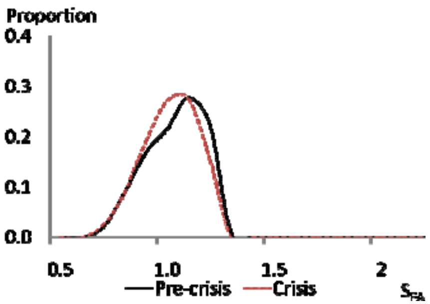
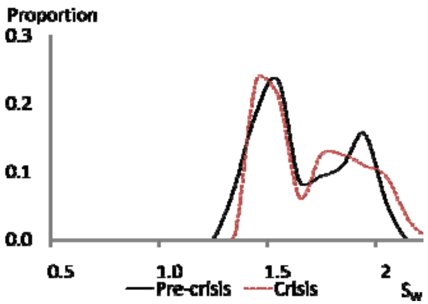


Figure 6: 2D representations of the S_{FA}, S_W, S^+_{TI}, S^-_{TI} descriptive statistics for US Treasuries in non-crisis and crisis periods

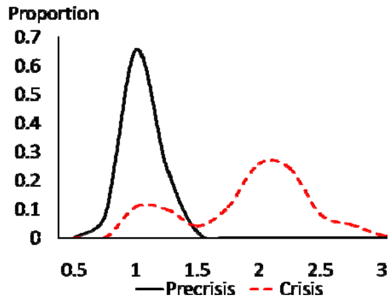
S_{FA}
2.1≤p≤6, k=2,3



S_W
1≤p≤1.75, k=2,3



S^+_{TI}
2.1≤p≤6, k=2,3



S^-_{TI}
2.1≤p≤6, k=2,3

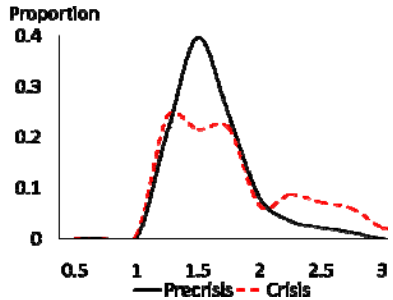


Figure 7: S, S+ and S- statistics for 2, 5, 10 and 30 year maturity bonds for the case of k=2 (top panel) and k=3 (bottom panel), $2.1 \leq p \leq 6$