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Sovereign risk and the effects of fiscal retrenchment in deep recessions

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Abstract

We analyze the effects of government spending cuts on economic activity in an environment of severe fiscal strain, as reflected by a sizeable risk premium on government debt. Specifically, we consider a "sovereign risk channel," through which sovereign default risk spills over to the rest of the economy, raising funding costs in the private sector. Our analysis is based on a variant of the model suggested by Cúrdia and Woodford (2009). It allows for costly financial intermediation and inter-household borrowing and lending in equilibrium, but maintains the tractability of the baseline New Keynesian model. We show that, if monetary policy is constrained in offsetting the effect of higher sovereign risk on private-sector borrowing conditions, the sovereign risk channel exacerbates indeterminacy problems: private-sector beliefs of a weakening economy can become self-fulfilling. Under these conditions, fiscal retrenchment can limit the risk of macroeconomic instability. In addition, if fiscal strain is very severe and monetary policy is constrained for an extended period, fiscal retrenchment may actually stimulate economic activity.

Keywords: Fiscal consolidation, Monetary policy, Zero lower bound, Risk premium, Sovereign risk channel

JEL-Codes: E62, E52, E32

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1 Introduction

In the wake of the global financial crisis, public debt in many industrialized countries has risen to such levels that fiscal retrenchment cannot be avoided. Standard models and evidence from vector autoregressions suggest that this retrenchment will weigh on short-term growth.[1] In fact, to the extent that monetary policy is constrained by the zero lower bound on interest rates (ZLB, henceforth), the headwinds from fiscal tightening could be even more severe (Christiano et al. 2011 and Woodford 2011).[2] Common wisdom, therefore, holds that retrenchment should be delayed until the economy has fully recovered (for example, Corsetti, Kuester, Meier and Müller 2010).

However, the sovereign risk premium has been rising sharply in several countries, causing policy-makers to start fiscal tightening even as private demand remains weak. What are the likely consequences for economic activity? In the present paper, we assess this question quantitatively, starting from the observation that strains in sovereign funding tend to spill over into private credit markets.[3] Because of such spillovers, rising sovereign indebtedness can negatively affect economic activity through its effects on interest rates faced by firms and households. Via this channel, fiscal retrenchment upfront can help improve credit conditions in the broader economy, thereby counteracting the otherwise contractionary effects of lower public spending.

Recent developments in Europe provide evidence in support of such a "sovereign risk channel." The panels in Figure 1 display time-series data on credit default swap (CDS) spreads for sovereign debt and non-financial corporate debt.[4] The figure focuses on two sets of euro area countries: those with relatively low sovereign spreads (left panel) and those with relatively

[1] Hall (2009) surveys the relevant empirical research that predates the recent global financial crisis and finds that most studies report government spending multipliers on output in the range of 0.5 to 1.0. The predictions of standard business cycle models fall in a similar range.

[2] Time-series evidence also indicates that multiplier effects tend to be larger during recessions (Auerbach and Gorodnichenko 2010) and during financial crises (Corsetti, Meier and Müller 2010).

[3] This is prominently embedded in the notion of a "sovereign ceiling." In a strict interpretation, the sovereign ceiling posits that no debtor in a given country can have a better credit quality than the government, a primary reason being the latter's capacity to extract private-sector resources through taxation. In reality, some authors, including Durbin and Ng (2005), have documented exceptions to this rule, notably for firms with substantial export earnings or close links to foreign firms. Even then, however, sovereign and corporate bond yields comove significantly (see, for instance, the literature review in Cavallo and Valenzuela (2007) or Harjes (2011)). In the context of the global financial crisis both the International Monetary Fund (2010*a*) and the European Central Bank (2010) have stressed that government bond yields typically have a strong influence on domestic corporate bond yields.

[4] A similar set of charts was first provided in International Monetary Fund (2011).

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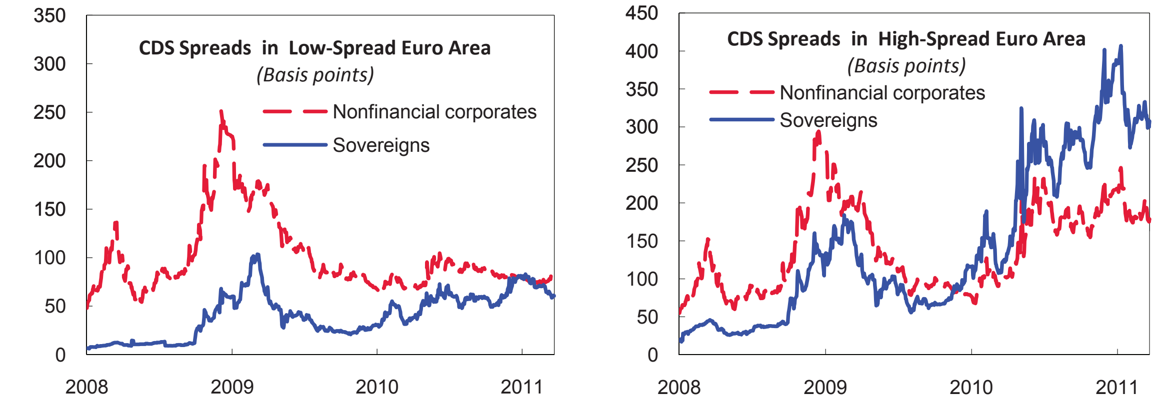
Figure 1: Sovereign and nonfinancial corporate CDS spreads



Notes: 5-year CDS spreads in high-spread and low-spread euro area countries, as well as for nonfinancial corporations headquartered there. Low-spread euro area includes Austria (number of firms in our sample: 1), Finland (1), France (24), Germany (18), and Netherlands (8). High-spread euro area includes Belgium (number of firms: 1), Greece (1), Ireland (0), Italy (4), Portugal (2), and Spain (4). The corporations in our sample are the constituents of the Itraxx Europe index. The same relative weights are adopted for the sovereign and corporate index series. For example, of the 52 firms in the low-spread euro area sample, 24 are headquartered in France. As a result, in the sovereign low-spread euro area series, France has a weight of 24/52. Data sources: Bloomberg; Markit.

high sovereign spreads (Belgium, Greece, Ireland, Italy, Portugal, and Spain).[5] The series display substantial comovement, particularly in countries that face fiscal strain (right panel). For the time period shown, the daily correlation between corporate and sovereign CDS spreads in high-spread countries is 0.71. For the low-spread countries, it is lower, but still significantly positive at 0.36 percent.

In this paper, we explore the implications of the sovereign risk channel building on the model proposed by Cúrdia and Woodford (2009). This allows for household heterogeneity, as private agents engage in borrowing and lending via financial intermediaries. In our variant of the model private credit spreads rise with sovereign risk because strained public finances raise the costs of financial intermediation. While this is not the only possible way of envisioning spillovers through the sovereign risk channel, it allows for a tractable representation within a simple variant of the canonical New Keynesian model. Consequently, we are in a position to complement our numerical results with analytical solutions for interesting special cases.

Our formal analysis of the sovereign risk channel gives rise to two distinct sets of results. Both are related to the fact that higher sovereign risk dampens aggregate demand, unless

[5] We focus on evidence for the euro area in order to control for monetary policy. Monetary policy is a key factor in determining the strength of the sovereign risk channel according to our analysis below.

monetary policy manages to offset the effect that sovereign risk has on private-sector funding costs. Offsetting sovereign risk would typically involve a cut in the policy rate. Yet the normal operation of monetary policy may be hampered when nominal rates are near zero. Our first finding is that under these circumstances sovereign risk may give rise to equilibrium indeterminacy. The reason is that private-sector beliefs about a weakening economy can become self-fulfilling. Specifically, a pessimistic shift in expectations implies an upward revision of the projected government deficit. This causes a higher risk premium on public debt and, through the sovereign risk channel, on private debt as well. Higher private funding costs, in turn, slow down activity, thus validating the initial adverse shift in expectations. Under normal circumstances, this scenario can arguably be averted by the central bank's commitment to appropriately adjust the policy rate. To the extent that monetary policy is constrained, however, expectations may become self-fulfilling, especially when sovereign risk is already high. Contrary to conventional wisdom, we therefore find that in the presence of severe fiscal strain, expectations of a pro-cyclical spending response, that is, fiscal tightening, can help to ensure determinacy.

Our second set of results concerns the sign and the size of the government spending multiplier. We find that for reasonable parameterizations the presence of a sovereign risk channel reduces the spending multiplier. A fiscal retrenchment may therefore have less adverse effects on economic activity than in the absence of sovereign risk. Quantitatively, however, the role of the sovereign risk channel is of limited importance even at high levels of public debt—provided that monetary policy is unconstrained and able to offset changes in the sovereign risk premium. By contrast, if public debt and hence sovereign risk is high, *and* monetary policy is constrained by the ZLB for an extended period, we find that fiscal retrenchment can stimulate economic activity, that is, the government spending multiplier turns negative.

Our results thus provide a fresh perspective on the "expansionary effects" of fiscal contractions that have been emphasized in a prominent study of Ireland and Denmark during the 1980s by Giavazzi and Pagano (1990).[6] In order to rationalize expansionary consolidations, theoretical accounts have often focused on the "expectations view," whereby immediate fiscal consolidation triggers a shift in expectations regarding the long-run level of spending, and

[6]The evidence on expansionary consolidations remains controversial. For a positive assessment, see, for example, Alesina and Perotti (1995) or Alesina and Ardagna (2010). A skeptical view is provided by International Monetary Fund (2010*b*). Perotti (2011) provides a reassessment.

thus a downward revision of the anticipated tax burden (Bertola and Drazen 1993, Sutherland 1997, and Perotti 1999).[7] However, Giavazzi and Pagano also stress that monetary policy may have played an important role. Specifically, the consolidations in Ireland and Denmark were accompanied by credible exchange rate pegs, which, arguably, led to declining country risk premiums and lower real interest rates. More recently, Erceg and Lindé (2010*a*) have analyzed government spending cuts in a model of a currency union where country risk is a function of the state of public finances.[8]

As a caveat we emphasize that the present paper is not meant to add to the theory of sovereign default. Following Eaton and Gersovitz (1981), a number of authors, including Arellano (2008) and Mendoza and Yue (2010), have recently modeled default as a strategic decision of a sovereign that balances the gains from forgone repayment against the costs of exclusion from international credit markets. In equilibrium this implies that the probability of default increases in the level of debt. In order to maintain the tractability of our model for business cycle analysis, we impose such a relationship without explicitly modeling a strategic default decision.[9] In the same vein, the current paper imposes a sovereign risk channel in order to explore its role for fiscal policy transmission, but it leaves a richer theoretical account of the underlying mechanism for future research. In particular, a straightforward yet crucial assumption in our analysis is that there are limits to credible commitment on the part of fiscal policymakers — otherwise, there would be no risk premium in the first place, and delaying retrenchment until the economy is on a firm recovery path would likely remain preferable.

The remainder of the current paper is structured as follows. Section 2 describes the model economy and presents our calibration. Sections 3 and 4 report analytical results and results from model simulations, respectively. Section 5 concludes.

[7] Bertola and Drazen (1993) analyze a neoclassical endowment economy where trigger points for fiscal adjustment may alter the comovement between private consumption and government spending, depending on the level of debt. Sutherland (1997) focuses on tax cuts. Perotti (1999) investigates government spending and taxes in a model with several frictions.

[8] In simulations, they find that output always falls in the initial periods of a persistent spending cut but that the effects on output may eventually turn positive during the dynamic adjustment process. For high levels of debt we obtain even stronger results—notably positive impact effects—as we consider a wider range of parameterizations and also allow for a nonlinear relationship between the risk premium and the level of public debt.

[9] Specifically, we link the sovereign risk premium to the expected path of public debt (or, alternatively, future fiscal deficits). We thereby abstract from a number of other factors that may also affect the markets' assessment of sovereign risk, such as the quality of fiscal institutions or the composition of the investor base for government bonds.

2 The model

We analyze the effects of fiscal retrenchment within a variant of the New Keynesian model of a closed economy. We are particularly interested in analyzing how changes in fiscal policy can affect private-sector borrowing conditions through the implied changes in sovereign risk. We therefore account for the possibility that private-sector borrowing and lending take place in equilibrium. In order to do so, we rely on the framework developed by Cúrdia and Woodford (2009) (CW, henceforth), which gives rise to an interest rate spread within an otherwise standard New Keynesian model. The spread emerges as a result of heterogeneity among households and because of costly financial intermediation. By assuming asymptotic risk sharing, CW are able to maintain the tractability of the New Keynesian baseline model. We add to their model a slightly richer specification of fiscal policy and allow the fiscal position to affect financial intermediation. In the following we briefly outline the model and stress the instances in which we depart from the original CW formulation.

2.1 Households

The economy is populated by a unit measure of households indexed by $i \in [0, 1]$. Household i is of one of two types, indexed by superscript $\tau_t(i) \in \{b, s\}$. In equilibrium, households of type $\tau_t(i) = b$ will be "borrowers" and households of type s will be "savers." Infrequently, households change their type. In each period, a household's probability of remaining its current type is given by $\delta \in (0, 1)$. With probability $1 - \delta$, the household draws a new type. With probability π_b the household will be a borrower, with probability $\pi_s = 1 - \pi_b$ the household will be a saver. The objective of household $i \in [0, 1]$ is given by

$$E_0 \sum_{t=0}^{\infty} (e_t \beta^t) \left[\frac{(\xi^\tau)^{\sigma_\tau^{-1}} [c_t(i)]^{1-\sigma_\tau^{-1}}}{1 - \sigma_\tau^{-1}} - \frac{\psi_\tau}{1+\nu} h_t(i)^{1+\nu} \right],$$

where $c_t(i)$ is an aggregate of household expenditures:

$$c_t(i) = \left[\int_0^1 c_t(j, i)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}; \quad \theta > 1. \tag{1}$$

Here $c_t(j, i)$ is a differentiated output good produced by firm $j \in [0, 1]$. $h_t(i)$ denotes hours worked by the household. e_t is a unit-mean shock to the time-discount factor $\beta \in (0, 1)$ and

$\xi^\tau, \sigma_\tau, \psi_\tau$ and ν are positive parameters.

Households are able to insure against idiosyncratic risk through state-contingent contracts. Yet the resulting transfer payments are assumed to occur only infrequently, namely only in those periods in which a household is assigned a new type. Meanwhile households may borrow or save through financial intermediaries. The beginning-of-period wealth of household i is given by

$$A_t(i) = [B_{t-1}(i)]^+(1+i^d_{t-1})+[B_{t-1}(i)]^-(1+i^b_{t-1})+(1-\vartheta_t)B^g_{t-1}(i)(1+i^g_{t-1})+D^{int}_t+T_t(i)+T^c_t. \quad (2)$$

Here $[B_{t-1}(i)]^+$ denotes deposits at financial intermediaries at the end of the previous period, which earn the deposit rate i^d_{t-1}; $[B_{t-1}(i)]^-$ denotes debt at financial intermediaries that charge the borrowing rate i^b_{t-1}. In equilibrium, household i is either borrowing or saving. In the case where it is saving, the household may also hold government debt $B^g_{t-1}(i) \geq 0$.

We depart from CW by assuming that government debt is not riskless: in any period, the government may honor its debt obligations, in which case $\vartheta_t = 0$, or it may partially default, in which case $\vartheta_t = \vartheta_{\text{def}}$, with $\vartheta_{\text{def}} \in [0, 1)$ being the size of the haircut. i^g_{t-1} is the notional interest rate on government debt. D^{int}_t are profits from competitive financial intermediaries that are distributed across households in a lump-sum manner. $T_t(i)$ denotes transfers resulting from state-contingent contracts (which are zero for those households that do not change their type and are therefore temporarily without access to the payoff scheme implied by asymptotic risk sharing). T^c_t is a lump-sum transfer that, in case of a sovereign default, compensates bond holders for losses associated with the sovereign default. Yet the payment is not proportional to the size of an individual's holdings of government debt (see Schabert and van Wijnbergen (2008) for a similar setup). This assumption along with the risk of a haircut drives a wedge between the risk-free rate, i^d_t, and the interest rate on sovereign debt, i^g_t.

The end-of-period wealth of household i given by

$$B_t(i) = A_t(i) - P_t c_t(i) + (1 - \tau^w_t)P_t w_t h_t(i) + D_t - T^g_t. \quad (3)$$

P_t denotes the consumption price index, τ^w_t is the labor tax rate, and w_t is the economy-wide real wage rate; D_t are profits by intermediary goods producers and $-T^g_t$ are lump-sum transfers by the government.

Assuming identical initial wealth for all households, state-contingent contracts ensure that post-transfer wealth is identical for all households that are selected to change their type. It is given by

$$A_t = [d_{t-1}(1+i^d_{t-1}) + (1-\vartheta_t)b^g_{t-1}(1+i^g_{t-1}) - b_{t-1}(1+i^b_{t-1})]P_{t-1} + D^{int}_t + T^c_t, \tag{4}$$

where b^g_t denotes government debt in real terms. d_t denotes aggregate savings deposited with intermediaries and b_t denotes aggregate private borrowing, both in real terms. The latter evolves according to

$$\begin{aligned} b_t &= \delta b_{t-1}(1+\omega_{t-1})(1+i^d_{t-1})/\Pi_t - \pi_b\omega_t b_t + \pi_b\left[\delta b^g_{t-1}(1+i^g_{t-1})/\Pi_t - b^g_t\right] \\ &\quad +\pi_b\pi_s[(c^b_t - c^s_t) - (1-\tau^w_t)(w_t h^b_t - w_t h^s_t)], \end{aligned} \tag{5}$$

Intuitively, the accumulation of debt depends on four terms. The first term is the last period's private debt level times interest (for those households that do not change their type). The second term, -$\pi_b\omega_t b_t$, is the gain accruing to borrowing households from fraudulent loans (discussed below). The third term captures whether sovereign indebtedness (suitably adjusted for the change in household types) falls. In order to reduce sovereign indebtness, current taxes need to be relatively high, which increases the need for borrowing by borrowers. Alternatively, if sovereign indebtedness does fall, so that $\left[\delta b^g_{t-1}(1+i^g_{t-1})/\Pi_t - b^g_t\right] > 0$, more resources are made available by savers to borrowers since savers resort more to private sources for storing value. The last term, on the second line, captures the difference in consumption levels relative to the difference in wage income across household types.

Turning to the intertemporal consumption decisions, note that, as a result of asymptotic risk sharing, all households of a specific type have a common marginal utility of real income, λ^τ_t, and choose the same level of expenditure:

$$c^s_t = \xi^s(\lambda^s_t)^{-\sigma_s} \tag{6}$$

$$c^b_t = \xi^b(\lambda^b_t)^{-\sigma_b}. \tag{7}$$

The optimal choices regarding borrowing from and lending to intermediaries, as well as to the government, are then governed by the following Euler equations

$$e_t\lambda_t^s = \beta E_t\left[e_{t+1}\frac{1+i_t^d}{\Pi_{t+1}}\left\{(1-\delta)\pi_b\lambda_{t+1}^b + [\delta+(1-\delta)\pi_s]\lambda_{t+1}^s\right\}\right], \tag{8}$$

$$e_t\lambda_t^s = \beta E_t\left[e_{t+1}\frac{(1-\vartheta_{t+1})(1+i_t^g)}{\Pi_{t+1}}\left\{(1-\delta)\pi_b\lambda_{t+1}^b + [\delta+(1-\delta)\pi_s]\lambda_{t+1}^s\right\}\right], \tag{9}$$

$$e_t\lambda_t^b = \beta E_t\left[e_{t+1}\frac{1+i_t^b}{\Pi_{t+1}}\left\{(1-\delta)\pi_s\lambda_{t+1}^s + [\delta+(1-\delta)\pi_b]\lambda_{t+1}^b\right\}\right]. \tag{10}$$

Optimal labor supply by households, in turn, is given by

$$h_t^s = \left(\frac{\lambda_t^s}{\psi_s}(1-\tau_t^w)w_t\right)^{1/\nu}, \tag{11}$$

$$h_t^b = \left(\frac{\lambda_t^b}{\psi_b}(1-\tau_t^w)w_t\right)^{1/\nu}. \tag{12}$$

Across household types, average labor supply, $h_t = \pi_b h_t^b + (1-\pi_b)h_t^s$, is given by

$$h_t = \left(\frac{\Lambda_t}{\psi}(1-\tau_t^w)w_t\right)^{1/\nu}, \tag{13}$$

where

$$\Lambda_t := \psi\left[\pi_b\left(\frac{\lambda_t^b}{\psi_b}\right)^{1/\nu} + \pi_s\left(\frac{\lambda_t^s}{\psi_s}\right)^{1/\nu}\right]^{\nu} \tag{14}$$

and $\psi^{-1/\nu} = \pi_b\psi_b^{-1/\nu} + \pi_s\psi_s^{-1/\nu}$. Finally, for future reference we define

$$\lambda_t = \pi_b\lambda_t^b + (1-\pi_b)\lambda_t^s \tag{15}$$

as the average marginal utility of real income across types.

2.2 Financial intermediaries

Saving and borrowing across households of different types takes place through perfectly competitive financial intermediaries. As in CW, we assume that an interest rate spread emerges, because financial intermediation requires resources, $\Xi_t b_t$, and because in each period a fraction of loans, χ_t, cannot be recovered, irrespective of the characteristics of borrowers (due to, e.g., fraud). Moreover, deposits, d_t, are assumed to be riskless and intermediaries are assumed to collect the largest quantity of deposits that can be repaid from the proceeds of the loans that it originates, that is, $(1+i_t^d)d_t = (1+i^d)b_t$. The cash flow in period t of a financial intermediary is thus given by $d_t - b_t - \chi_t b_t - \Xi_t b_t$. Using ω_t to define the spread between lending and deposit rates, we have

$$1+\omega_t = \frac{1+i_t^b}{1+i_t^d}. \tag{16}$$

Substituting $d_t = (1+\omega_t)b_t$, and choosing b_t to maximize the profits of the intermediary yields the first-order condition for loan origination

$$\omega_t = \chi_t + \Xi_t. \tag{17}$$

In departing from CW, we assume that either χ_t or Ξ_t depends on sovereign risk—to capture increased strain on the financial system and, hence, the increased difficulties in monitoring and enforcing loan contracts in an economy under fiscal strain. Conceptually related is the notion that in case of sovereign default, the government diverts funds from the repayment of borrowers, see Mendoza and Yue (2010).

Costs $\chi_t b_t$ and $\Xi_t b_t$ differ in that only the latter are assumed to enter the economy's resource constraint. For the linearized version of the model, used in Section 3, we let loan origination costs be covered by $\chi_t > 0$, and set $\Xi_t = 0$, which facilitates deriving analytical results. For the dynamic simulations in Section 4 we set $\chi_t = 0$ and let $\Xi_t > 0$. Specifically, we assume that either

$$\chi_t = \chi_\psi[(1+i_t^g)/(1+i_t^d)]^{\alpha_\psi} - 1 \text{ and } \Xi_t = 0, \tag{18}$$

or

$$\chi_t = 0 \text{ and } \Xi_t = \chi_\psi[(1+i_t^g)/(1+i_t^d)]^{\alpha_\psi} - 1, \tag{19}$$

where parameter $\chi_\psi > 0$ is used to scale the private spread in the steady state, and α_ψ measures the strength of the spillover of the (log) sovereign risk premium to the (log) private risk premium. Finally, transfers from intermediaries to households include loans that are not recovered by the intermediaries such that $D_t^{int} = P_t(\omega_t b_t - \Xi_t\, b_t)$.

2.3 Firms

There is a continuum of firms $j \in [0,1]$, each of which produces a differentiated good on the basis of the following technology

$$y_t(j) = z_t h(j)^{1/\phi}, \tag{20}$$

where z_t is an aggregate productivity shock. In each period only a fraction $(1-\alpha)$ of firms is able to reoptimize its prices. Firms that do not reoptimize adjust their price by the steady-state rate of inflation, Π. Prices are set in period t to maximize expected discounted future profits.[10] The resulting first-order condition for a generic firm that adjusts its price, P_t^*, is

$$\left(\frac{P_t^*}{P_t}\right)^{1+\theta(\phi-1)} = \frac{K_t}{F_t}, \tag{21}$$

and

$$K_t = \lambda_t e_t \mu^p \phi w_t \left(\frac{y_t}{z_t}\right)^{\phi} + \alpha\beta E_t\left[\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta\phi} K_{t+1}\right], \tag{22}$$

$$F_t = \lambda_t e_t y_t + \alpha\beta E_t\left[\left(\frac{\Pi_{t+1}}{\Pi}\right)^{(\theta-1)} F_{t+1}\right], \tag{23}$$

where $\mu^p = \theta/(\theta-1)$. The law of motion for prices (inflation) is given

$$1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1} = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{1-\theta}. \tag{24}$$

For future reference it is also useful to define price dispersion $\Delta_t := \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta\phi} dj$, which evolves as follows

$$\Delta_t = \alpha\Delta_{t-1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta\phi} + (1-\alpha)\left(\frac{1-\alpha\left(\Pi_t/\Pi\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}. \tag{25}$$

Finally, profits distributed to households are given by $D_t = \int_0^1 P_t(j)y_t(j) - P_t w_t h_t(j)dj$. Or, in equilibrium, $D_t = P_t\left(y_t - w_t\left(y_t/z_t\right)^{\phi}\Delta_t\right)$.

[10] Future nominal profits are discounted with the factor $(\alpha\beta)^{T-t}\frac{\lambda_T}{\lambda_t}\frac{P_t}{P_T}$, taking into account that demand for product j is given by the demand function $y_t(j) = y_t(P_t(j)/P_t)^{-\theta}$, where $P_t(j)$ denotes the price of good j and y_t is aggregate output.

2.4 Government

Real government debt evolves as follows

$$b_t^g = (1-\vartheta_t)\frac{b_{t-1}^g(1+i_{t-1}^g)}{\Pi_t} + g_t + \frac{T_t^c}{P_t} - \frac{T_t^g}{P_t} - \tau_t^w w_t h_t,$$

where g_t denotes government spending. Below we will consider alternative assumptions regarding the law of motion for government spending. As is customary, throughout the paper, we assume that the expenditure share of each particular differentiated good in government spending is the same as the share of that good in private consumption. By assumption, transfers T_t^c ensure that a sovereign default is neutral *ex post* in regard to any distributional consequences and the debt level. That is, under our assumptions, a sovereign default does not automatically ease the degree of fiscal strain. In particular, we set

$$T_t^c = P_t \vartheta_t \frac{b_{t-1}^g(1+i_{t-1}^g)}{\Pi_t}.$$

The consolidated government flow budget constraint is thus given by

$$b_t^g = \frac{b_{t-1}^g(1+i_{t-1}^g)}{\Pi_t} + g_t - \frac{T_t^g}{P_t} - \tau_t^w w_t h_t. \tag{26}$$

Letting

$$tr_t = \tau_t^w w_t h_t + T_t^g / P_t \tag{27}$$

be the part of taxes that is related to the business cycle and to stabilization policy, we assume

$$(tr_t - t) = \left[\phi_{T,y}(y_t - y) + \phi_{T,b^g}(b_{t-1}^g - b^g)\right],\ \phi_{T,y} \geq 0,\ \phi_{T,b^g} > 0. \tag{28}$$

Throughout the paper, we assume that $\phi_{T,b}$ is large enough so as to eventually stabilize public debt.[11]

While *actual* default is neutral in the sense described above, the *probability* of a default is crucial for the pricing of government debt (i_t^g). And this probability of default – through financial intermediation – does matter for real activity.[12] Yet a fully specified model of

[11] We will also, for a large part, assume that the labor tax rate remains constant, $\tau_t^w = \tau^w$, and will be explicit when we consider simulations in which that is not going to be the case.

[12] This implication of our setup is in line with evidence reported by Yeyati and Panizza (2011). Investigating

sovereign default is beyond the scope of the present paper. In this regard the literature has pursued two distinct approaches. First, following Eaton and Gersovitz (1981), Arellano (2008) and others have modeled default as a strategic decision of the sovereign. Second, and more recently, Bi and Leeper (2010) and Juessen et al. (2011), consider default as the consequence of the government's inability to raise the funds necessary to honor its debt obligations. Under both approaches, the probability of sovereign default is tightly and nonlinearly linked to the level of public debt.

In the current paper we operationalize sovereign default by appealing to the notion of a fiscal limit in a manner similar to Bi and Leeper (2010). Whenever the debt level rises above the fiscal limit, a default will occur. The fiscal limit is determined stochastically capturing the uncertainty that surrounds the political process in the context of sovereign default. Specifically, we assume that in each period the limit will be drawn from a generalized beta distribution with parameters α_{b^g}, β_{b^g}, and $\overline{b}^{\mathrm{g,max}}$. As a result, the *ex ante* probability of a default, p_t, at a certain level of sovereign indebtedness, b_t^g, will be given by the cumulative distribution function of the (generalized) beta distribution:

$$p_t = F_{\mathrm{beta}}\left(\frac{b_t^g}{4y}\frac{1}{\overline{b}^{\mathrm{g,max}}}; \alpha_{b^g}, \beta_{b^g}\right). \tag{29}$$

Note that $\overline{b}^{\mathrm{g,max}}$ denotes the upper range of the support for the debt level in terms of the debt-to-GDP ratio. Since we keep the size of the haircut in case of a default constant, we have

$$\vartheta_t = \begin{cases} \vartheta_{\mathrm{def}} & \text{with probability } p_t, \\ 0 & \text{with probability } 1 - p_t. \end{cases} \tag{30}$$

Turning, last, to monetary policy, throughout the paper we assume that monetary policy follows a Taylor-type interest rate rule that also seeks to insulate aggregate economic activity from fluctuations in spreads, at least to some degree. In particular, we assume:

$$\log(1 + i_t^{d,*}) = \log(1 + i^d) + \phi_\Pi \log(\Pi_t/\Pi) - \phi_\omega \log((1 + \omega_t)/(1 + \omega)). \tag{31}$$

Here, $i_t^{d,*}$, marks the target level for the deposit rate i_t^d, and $\phi_\Pi > 1$, $\phi_\omega > 0$. In deep

output growth in a large number of episodes of sovereign defaults on the basis of quarterly data, they find that the output costs of a default materialize in the run-up to defaults rather than at the time when a default actually takes place.

recessions, the target level and the actual deposit rate can diverge. The reason is that in implementing rule (31), the central bank relies on steering the riskless interest rate i_t^d, which cannot fall below zero. Therefore, $i_t^d = i_t^{d*}$ can be implemented provided that $i_t^d \geq 0$. Otherwise, $i_t^d = 0$. As a result, an increase in the spread ω_t cannot be offset if monetary policy is constrained in lowering the policy rate.[13]

2.5 Market clearing and equilibrium

Good market clearing requires

$$y_t = \int_0^1 c_t(i)di + g_t + \Xi\, b_t = \pi_b c_t^b + \pi_s c_t^s + g_t + \Xi\, b_t \tag{32}$$

The total supply of output is given by

$$y_t \Delta_t^{1/\phi} = z_t h_t^{1/\phi}. \tag{33}$$

In order to characterize the equilibrium, we use equations (5)-(15), which characterize the solution to the household problem; equations (16)-(19), which characterize financial intermediation; equations (21)-(25), which characterize optimal price setting behavior; equations (26) - (30), which characterize the behavior of fiscal policy, and the assumption about the evolution of labor taxes, the interest rate target rule (31) and the lower-bound constraint, and finally the good market clearing conditions (32) and (33). For given exogenous realizations of $\{e_t, g_t, z_t\}$ these equations pin down a sequence for the endogenous variables

$$\Big\{ b_t, b_t^g, c_t^b, c_t^s, \chi_t, \Delta_t, F_t, h_t, h_t^b, h_t^s, i_t^b, i_t^d, i_t^{d,*}, i_t^g, K_t, \lambda_t, \lambda_t^b, \lambda_t^s, \Lambda_t,$$
$$\omega_t, p_t, P_t^*/P_t, \Pi_t, \tau_t^w, T_t^g/P_t, tr_t, \vartheta_t, w_t, \Xi_t, y_t \big\}.$$

[13] Here we focus on a simple representation of monetary policy. In the current model environment, monetary policy could – in principle – take a much more complicated form. For example, monetary policy in a lower bound situation could promise low future real rates to help the economy ease out of the lower bound situation; see Eggertsson and Woodford (2003). This would not only increase output relative to the current interest rate rule (31), but it would also raise tax revenues and would therefore, to some extent, ease the fiscal strain. The question of the extent to which central banks can credibly engage in such complicated forward guidance is not settled, however.

2.6 Calibration

In order to solve the model numerically, we assign parameter values on the basis of observations for U.S. data and on the basis of the relationship between sovereign risk, private-sector spreads, and the debt level across a range of countries. A time period in the model is one quarter.

In regard to monetary policy, we assume an average inflation rate of 2 percent per year. The coefficient on inflation in the Taylor rule is set to a customary value of $\phi_\Pi = 1.5$. We entertain different values for parameter ϕ_ω, the response of monetary policy to the interest spread below. We will discuss the parameterization in the respective sections.

The steady-state level of government spending (consumption and investment) relative to the size of GDP is $g/y = 0.2$. The level of gross public debt in the steady state is set to 60 percent of annual GDP. These values are broadly in line with U.S. averages over the last 20 years. In the baseline scenario, we set distortionary tax rates to zero and assume that the adjustment of taxes over the business cycle and in response to the debt level is achieved through lump-sum taxes. This assumption allows us to focus on the main channels of transmission in a transparent way while accounting for a feedback from economic activity to the fiscal outlook.[14] We assume that taxes react to debt sufficiently strongly ($\phi_{T,y}$ large enough) so as to ensure that the debt level remains bounded throughout and that $\phi_{T,y} = 0.34$. This value is reasonable for the U.S., but at the lower end of estimates for other OECD countries; compare Girouard and André (2005).[15]

With regard to the preference parameters, we set the curvature of the disutility of work to $\nu = 1/1.9$, in line with the arguments provided by Hall (2009) regarding plausible values for the Frisch elasticity of labor supply. We set an elasticity of demand of $\theta = 7.\overline{6}$ so as to generate a gross price markup of $\mu^p = 1.15$, which is in the range of customary values used in the literature. Finally, for our baseline scenario, we assume that the average intertemporal elasticity of substitution $\overline{\sigma} = c/y$, where $\overline{\sigma} := \pi_b \cdot (c^b/y) \cdot \sigma_b + \pi_s \cdot (c^s/y) \cdot \sigma_s$. Had the model a representative household, this would correspond to the case of log-utility. Further, we

[14] Below we explore the sensitivity of our results and also consider a distortionary tax rate on labor.

[15] These authors follow the OECD's disaggregated approach, distinguishing four sources of tax revenues: personal income tax, Social Security contributions, corporate income tax and indirect taxes; in addition the estimates take into account unemployment-related transfers. For all five categories, the output elasticity is decomposed into i) the tax-base elasticity of a particular revenue/expenditure type and ii) the output elasticity of the tax/expenditure base in question. These components are quantified on the basis of different estimation strategies and combined to compute the output semi-elasticity of the budget.

assume that aggregate hours worked in the steady state are given by $h = 1/3$. We choose the relative values of the intertemporal elasticity of substitution of the two types of households (σ_b and σ_s) and of the scaling parameters of the disutility of work (ψ_b and ψ_s) such that the linearized model can be represented in the canonical three-equation New Keynesian format. This representation allows us to derive a number of analytical results in the next section. Importantly, under this calibration only the current value of the interest rate spread enters the dynamic IS-relationship and the New Keynesian Phillips curve. In addition, the evolution of output and inflation is independent of the level of private debt. Appendix F spells out in detail the conditions under which this representation is valid. Specifically, given the other parameter values, we set $\sigma_b/\sigma_s = 0.53$ and $\psi_b/\psi_s = 0.82$. We explore to sensitivity of our results with respect to these assumptions through numerical simulations in Section 4.

We target a ratio of private debt to annual GDP, $b/4y$, of 80 percent, in line with Great Moderation averages for the U.S. More precisely, the figure refers to nonfinancial, nonmortgage, nongovernment credit market debt outstanding recorded in the U.S. flow of funds accounts. The same target is used by Cúrdia and Woodford (2009). Along with the market clearing condition, this determines scaling parameters ξ^b and ξ^s. Next, as in Cúrdia and Woodford (2009), we assume that households change type on average every 40 quarters, giving $\delta = 0.975$. This implies that the average time during which a specific type is without access to payoff streams from asymptotic risk sharing is 10 years.

A central element in our calibration is the share of borrowers in the economy, π_b. This determines the share of economic activity that is affected by an increase in the spread and therefore deserves some discussion. One possible calibration would refer to the (U.S.) Survey of Consumer Finances. Averaging over the latest surveys (1998, 2001, 2004, and 2007), the share of U.S. families that hold some kind of debt is 76 percent; compare Aizcorbe et al. (2003) and Bucks et al. (2009). This suggests a value of $\pi_b = 0.76$, and of $\pi_s = 0.24$. However, loans secured by the primary residence make up a large share of that debt. This suggests that such a calibration for π_b might overstate the importance of borrowing and the related effect that an increase in borrowing spreads could have on economic activity. Another metric, also from the Survey of Consumer Finances, that is more directly related to the notion of "borrowers" and "savers" in our model is that on average 57 percent of families in the survey report that – over the year preceding each survey date – they have been spending less than their income, that is, they have saved. This suggests a value for π_b of $1 - 0.57$, or $\pi_b = 0.43$. That said, both of the aforementioned figures do not explicitly take into account the borrowing by firms in the economy (other than by single-owner firms). To the extent that households in our model own firms and also make the intertemporal decisions for these firms, any purely household-based measure of indebtedness is likely to underestimate the degree of indebtedness and thereby the importance of the borrowing spread. In particular, using the same measure of private borrowing as above (nonfinancial, nonmortgage, nongovernment credit market debt), 50 percent of private borrowing is accounted for by corporations rather than by households. In our baseline calibration, and in order to account for this, we set $\pi_b = (1 - 0.17) \cdot 0.43 + 0.17 \cdot 1 = 0.53$. This formula hypothetically divides households into consumption entities that have a certain share of indebtedness and investment entities, all of which have debt. In the calculations, 0.17 is the share of nonresidential private domestic fixed and inventory investment in private domestic economic activity.

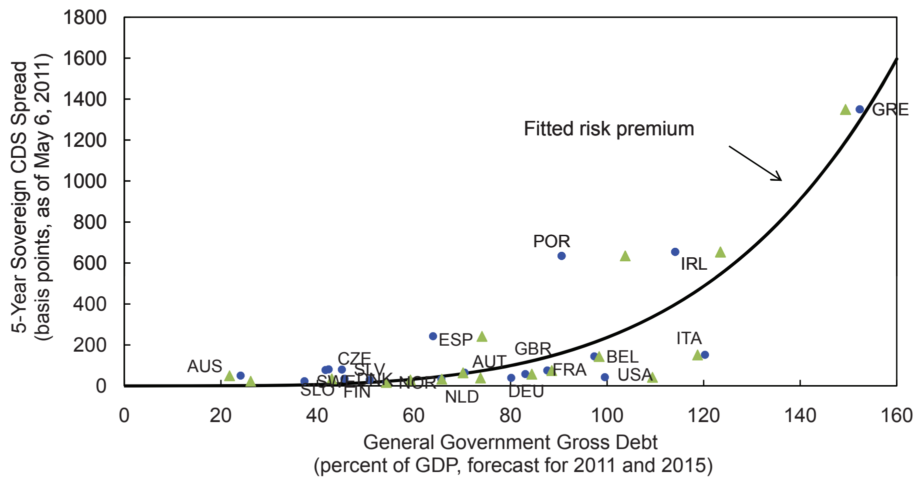
In regard to the normal spread between deposit and lending rates, we target a steady-state value of 2.1 percent (annualized), in line with commercial and industrial loan rate spreads in the Federal Reserve's Survey of Terms of Business Lending. This pins down parameter χ_ψ. The steady-state level for the central bank's target interest rate, i^d, is set to 4.5 percent annualized, from which the time discount factor, β, follows.

Next, in regard to production parameters, we set $\phi = 1$, implying a linear production function. We furthermore target a unit value for steady-state output, setting productivity, z, accordingly. We set parameter $\alpha = 0.9$ in order to generate a slope of the Phillips curve in line with the empirical evidence.[16]

Finally, it remains to determine the parameters that govern the spillover from sovereign risk premiums to private-sector spreads. Actual haircuts in case of a sovereign default show a large variation; see Panizza et al. (2009) and Moody's Investors Service (2011). $\vartheta_{\text{def}} = 0.5$ appears to be a reasonable average value. In regard to the specification of the fiscal limit, we seek to replicate the relationship between the sovereign risk premium and public debt shown in Figure 2. The figure plots CDS spreads of industrialized economies against the level of projected gross public debt (relative to GDP). The projections are taken from the

[16]Specifically, our parameterization implies a slope coefficient of $\kappa = 0.012$. Galí and Gertler (1999) report estimates for the slope of the Phillips curve, given by $(1 - \beta\theta)(1 - \theta)/\theta$, in the range between 0.007 and 0.047. More recently, Altig et al. (2010) report an estimate of 0.014.

Figure 2: Sovereign risk premia vs. debt



Notes: The figure shows 5-year sovereign CDS spreads for industrialized countries against forecasts for end of 2011 gross general government debt/GDP (blue circles) and end of 2015 debt to GDP (green triangles). The countries are Australia, Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, United Kingdom, United States. Note: Excludes Japan. The forecasts are taken from the IMF World Economic Outlook April/2011.

IMF World Economic Outlook in April 2011. The blue dots show projections for the end of 2011. For comparison, the figure also plots IMF forecasts for the debt to GDP ratio by the end of 2015. For the countries shown in the figure, CDS spreads are systematically higher the higher the level of projected gross public debt.[17] In fact, the risk premium appears to rise disproportionately as the debt level rises. We choose parameters $\alpha_{b^g} = 3.70$, $\beta_{b^g} = 0.54$, and $\overline{b}^{\mathrm{g,max}} = 2.56$ to match this empirical relationship. The black solid line displays the steady-state relationship between debt levels and the sovereign risk premium thus implied.

Regarding the spillovers from sovereign to private-sector risk, Figure 1 is suggestive of a sovereign risk channel that runs from sovereign spreads to spreads in the household and corporate sector. Of course, there may be other reasons for the observed comovement, too.

[17] For a systematic empirical analysis of the relationship between fiscal variables and yields on government bonds; see, among others, Reinhart and Sack (2000), Ardagna et al. (2007), Baldacci et al. (2008), Haugh et al. (2009), Baldacci and Kumar (2010), Laubach (2009) or Borgy et al. (2011). Ardagna et al. (2007) explicitly focus on possible nonlinearities in the relationship and find that bond rates rise disproportionately for very high levels of debt. Note, however, that sovereign CDS spreads may not only be driven by fiscal "fundamentals," but that these may compensate for factors other than default risk as well, from which we do abstract in the following. Rather, here we focus on the fact that high current and/or projected debt is consistently found to be a key determinant of government financing costs.

In the present paper, however, we abstract from these and interpret the comovement as caused by sovereign risk. In regard to α_ψ, for euro area sovereigns and a sample of large, publicly traded companies headquartered in these countries, Harjes (2011) finds that of a 100-basis-point increase in sovereign spreads, about 50 to 60 basis points are passed on to private firms. As our baseline, we therefore set $\alpha_\psi = 0.55$. For the simulations that we will show, we view this as a lower bound for two reasons: first, it is based on credit-spreads of companies that are large and therefore do have access to the international credit market. Indeed, many of the companies in the sample are internationally well-diversified. The spillover effects likely are larger for smaller – and less-diversified – companies that rely on local bank-based financing. Second, Figure 1 suggests that the comovement is considerably stronger in countries that face more fiscal strain than it is for countries with a more stable fiscal position. In that sense, the baseline value of $\alpha_\psi = 0.55$ may understate the strength of the sovereign-risk channel for highly indebted countries, and we also consider higher values as we move through the simulations in the paper.

3 Analytical results on the effects of fiscal retrenchment

We now turn to an analysis of the effects of fiscal retrenchment within the model outlined above. Our particular interest is in how the sovereign-risk channel affects the transmission of fiscal policy. A key aspect concerns the ability of monetary policy to offset the effect of sovereign risk on interest rates in the private sector. To capture this aspect we consider a scenario where monetary policy is possibly constrained by the ZLB. Before turning to simulation results for the full model, the current section focuses on a special case for which we are able to obtain analytical solutions. For this case, we assume that the probability of sovereign default depends on the expected primary deficit, rather than on the level of debt.

3.1 A special case of the model

In this section, we focus on a first-order approximation of the equilibrium conditions around the deterministic steady state. The aggregate equilibrium dynamics of the model can be represented by a variant of the New Keynesian Phillips curve and a dynamic IS-relationship.

The former relates inflation to expected inflation and output as well as government purchases:

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \kappa_y \tilde{y}_t - \kappa_g \tilde{g}_t, \tag{34}$$

where $\kappa_y = \kappa(\nu + \bar{\sigma}^{-1})$ and $\kappa_g = \kappa \bar{\sigma}^{-1}$, where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$.[18] In terms of notation, $\tilde{y}_t = y_t - y$, $\tilde{g}_t = g_t - g$, $\widehat{\Pi}_t = \log(\Pi_t/\Pi)$, where variables without a time subscript refer to a steady-state value.

The dynamic IS-relationship links output to real government spending and the effective real interest rate through

$$\tilde{y}_t - \tilde{g}_t = E_t \tilde{y}_{t+1} - E_t \tilde{g}_{t+1} - \bar{\sigma}\left[\widehat{i}_t^d + (\pi_b + s_\Omega)\,\widehat{\omega}_t - E_t \widehat{\Pi}_{t+1} + \Gamma_t\right], \tag{35}$$

where $\widehat{\omega}_t := \log((1+\omega_t)/(1+\omega))$, $\widehat{i}_t^d := \log((1+i_t^d)/(1+i^d))$, and $\Gamma_t := E_t \log(e_{t+1}) - \log(e_t)$. From the IS-relationship, it is clear that fluctuations in the private-sector spread can influence economic activity if these are not neutralized by monetary policy. The degree to which the private-sector spread, holding the policy rate constant, does affect economic activity in turn is determined by parameters $\pi_b + s_\Omega$. As discussed in Cúrdia and Woodford (2009), parameter $s_\Omega := \pi_b \pi_s (\sigma_b\, c_b/y - \sigma_s\, c_s/y)/\bar{\sigma}$ indicates the extent to which interest rate increases affect the aggregate demand by borrowers more adversely than that of savers. In our calibration, $c_b > c_s$ and $s_\Omega > 0$.

For monetary policy, equation (31) implies that in deviations from steady state

$$\widehat{i}_t^d = \max\{\phi_\pi \widehat{\Pi}_t - \phi_\omega \widehat{\omega}_t, -\log(1+i^d)\}. \tag{36}$$

For the analytical results, we focus on the case $\phi_\omega = (\pi_b + s_\Omega)$, so that in normal times the central bank fully sterilizes the effect of the sovereign risk premium on aggregate economic outcomes, as is borne out by the IS equation (35). Yet, monetary policy may not always be able to do so. In particular, in the following we assume that there are shocks to the time-discount factor, Γ_t, which reduce private expenditure and inflation by enough to push the policy rate to the ZLB. As a result monetary policy becomes constrained and unable to absorb an increase in the interest rate spread. In the following, we follow Christiano et al.

[18] Here, as in the following linearizations, we abstract from fluctuations in productivity, z_t.

(2011) and Woodford (2011) in assuming that these shocks follow a Markov structure: they persist into the next period with probability $\mu \in [0,1)$.[19] Given that there are no endogenous state variables in the special case of our model that we discuss in the current section, the expected duration of the recession, that is, the expected length of the ZLB episode, is given by $1/(1-\mu)$. Once the shock ceases to persist, the economy immediately reverts to the steady state.

Finally, as indicated above, we make a further simplifying assumption in this section that allows us to obtain analytical results. Namely, we assume that the probability of sovereign default – and thereby the sovereign-risk premium – depends on the expected primary deficit rather than on the level of public debt as in the full model. As a result, the interest rate spread depends on the expected deficit as well. In particular, we postulate a linearized relationship of the form

$$\widehat{\varpi}_t = \xi E_t(\tilde{g}_{t+1} - \phi_{T,y}\tilde{y}_{t+1}), \tag{37}$$

where, in order to ease the burden on notation, we have defined the spread that enters the IS-relationship over and above the riskless deposit rate as $\widehat{\varpi}_t := (\pi_b + s_\Omega)\widehat{\omega}_t$. Parameter $\xi \geq 0$ indicates the extent to which fiscal strain – as measured by primary deficits – spills over to private-sector spreads.

3.2 The size of the spillover

To appreciate our results below, it is useful to discuss the range of plausible values for ξ in equation (37), which — through a sequence of back-of-the-envelope calculations — links to the fundamental parameters of the model as follows. Let ξ' be the slope of the risk premium with respect to the deficit (or alternatively debt) at a specific debt level, evaluated in the steady state. Our assumptions regarding the sovereign spread in Section 2, in particular equation (29), imply that

$$\xi' = \alpha_\psi \frac{(\pi_b + s_\Omega)\,\vartheta_{\text{def}}}{1 - \vartheta_{\text{def}}\,F_{\text{beta}}\left(\frac{b^g}{4y}\frac{1}{\bar{b}^{\text{g,max}}};\,\alpha_{b^g},\beta_{b^g}\right)} \frac{1}{4y}\frac{1}{\bar{b}^{\,\text{g,max}}} f_{\text{beta}}\left(\frac{b}{4y}\frac{1}{\bar{b}^{\text{g,max}}};\,\alpha_b,\beta_b\right).$$

[19]Specifically, we assume a temporary increase in the effective discount factor, triggered by $0 < e_t = e_L < 1$ while at the ZLB, so $\Gamma_t = \mu\log(e_L) - \log(e_L) = -(1-\mu)\log(e_L) > 0$.

Table 1: Quantifying parameter ξ

debt/GDP	ξ'	ξ by length of ZLB (qtrs) 6	7	8
60 percent	0.0005	0.004	0.005	0.005
90 percent	0.0016	0.014	0.015	0.017
110 percent	0.0030	0.025	0.028	0.031
130 percent	0.0051	0.042	0.047	0.052
140 percent	0.0065	0.054	0.060	0.066
150 percent	0.0083	0.068	0.076	0.084

Notes: The table presents estimates for the slope of the average private interest rate with respect to the deficit, ξ, for different average lengths (in quarters) of the lower bound situation and for different debt/GDP ratios. The entries in the columns "ξ by length of ZLB (qtrs)" are based on the formula $\xi = \frac{1+\mu(1-\mu)}{\mu(1-\mu)}\,\xi'$ that is explained in detail in the main text and Appendix G.

The first column of Table 1 reports the resulting values for ξ' for alternative debt levels using the calibration of the fiscal limit distribution discussed in Section 2.6.

These values appear to be fairly small. It needs to be borne in mind, however, that the relationship in equation (37) links the interest rate spread and the expected deficit, whereas the full model implies a link between the interest rate spread and the expected level of debt — and therefore the accumulated deficits. The values for ξ' are thus likely to understate the size of the response of the interest rate spread to the fiscal situation. In particular, an appropriate mapping from the slope of the risk premium into the simplified model environment would appear to need to take into account the horizon over which deficits accumulate. The following expression is meant to capture this effect for empirically reasonable values of $\mu > 0.5$ (so the lower bound is expected to be binding for at least two periods):[20]

$$\xi = \frac{1+\mu(1-\mu)}{\mu(1-\mu)}\,\xi'. \tag{38}$$

The columns under "ξ by length of ZLB" of Table 1 report the corresponding values of ξ for different initial debt levels if the ZLB has an expected duration of six, seven or eight quarters. These calculations suggest that a value of ξ of about 0.1 cannot be ruled out if the initial level of debt is high and the recessionary shock is persistent. In particular, Figure 1 suggests that the spillovers can be notably stronger for countries that do face fiscal strain than for the average country. A bigger spillover parameter, α_ψ, would scale up linearly the entries in

[20] Appendix G presents a more detailed motivation of the formula.

Table 1.

3.3 Sovereign-risk channel and equilibrium determinacy

In our baseline scenario the level of government spending is determined exogenously. For this case, we find that the presence of a sovereign-risk channel alters the determinacy properties of the model while the ZLB is binding. In the following, we establish restrictions on parameters that ensure that the equilibrium is (locally) determinate.[21]

Proposition 1 *In the economy summarized by equations* (34) – (37)*, let* Γ_t *take on a positive value* $\Gamma > 0$ *in period zero, and remain such with probability* μ *in each subsequent period, until it reverts to* $\Gamma_t = 0$ *forever. Furthermore, let the value of* Γ_t *be large enough that the lower bound is binding initially. There is a unique bounded equilibrium if and only if*

$$a) \quad a < 1/(\beta\mu), \quad \text{and} \quad b) \quad (1-\beta\mu)(1-a) > \mu\bar{\sigma}\kappa_y,$$

where $a := \mu + \mu\xi\phi_{T,y}\bar{\sigma}$ *and* $\kappa_y := \kappa[\nu + 1/\bar{\sigma}]$.

Proof. See Appendix B. ■

In the absence of an endogenous risk premium, $\xi = 0$, as in Christiano et al. (2011) and Woodford (2011), condition a) is always satisfied. So there will be a unique bounded equilibrium if and only if condition b) holds. If $\xi = 0$, condition b) is given by $(1-\beta\mu)(1-\mu) > \mu\bar{\sigma}\kappa_y$. The previous literature has shown that the set of "fundamental" parameters for which this condition holds is the larger (i) the less persistent the lower bound situation (in our parameterization, the smaller μ), (ii) the lower the interest sensitivity of demand (the smaller $\bar{\sigma}$) and (iii) the flatter the Phillips curve (the smaller κ_y). Relative to these findings, our analysis shows that the range of parameters for which the equilibrium is determinate actually shrinks in the presence of a sovereign-risk channel. Namely, with $\xi > 0$, condition a) is violated if either the interest rate spread is sufficiently responsive to the deficit or if the tax revenue is

[21] Here we focus on local determinacy once the economy has reached the lower bound. Another strand of the literature examines global determinacy in the New Keynesian model and is concerned with preventing the economy from falling into a liquidity trap in the first place. Benhabib et al. (2002), for example, propose switching to a non-Ricardian fiscal policy. These mechanisms will rule out liquidity traps by making the low-inflation steady state fiscally unsustainable. Mertens and Ravn (2010) study the efficacy of fiscal policy in belief-driven equilibria.

sufficiently responsive to output ($\phi_{T,y}$ is large enough). Note that the same parameters are also key determinants for whether condition b) is satisfied.[22]

It is instructive to contrast this result for the baseline scenario with a situation where government spending adjusts endogenously to output while the economy is at the ZLB. The following proposition summarizes the conditions for the existence of a unique bounded equilibrium.

Proposition 2 *In the economy specified in Proposition 1, let government spending $\tilde{g}_t$ take on a value of $\tilde{g}_t = \varphi\tilde{y}_t$, when the economy is at the ZLB, and $\tilde{g}_t = 0$ otherwise. Suppose further that $\varphi < 1$. Define $a^* := \mu + \mu\xi\phi^*_{T,y}\bar{\sigma}^*$; $\kappa^*_y = \kappa_y - \varphi\kappa_g$; $\phi^*_{T,y} := \phi_{T,y} - \varphi$, and $\bar{\sigma}^* = \bar{\sigma}/(1-\varphi)$. There exists a unique bounded equilibrium if and only if:*

1. *with $a^* > 0$*

$$a)\quad a^* < 1/(\beta\mu), \quad \text{and} \quad b)\quad (1-\beta\mu)(1-a^*) > \mu\bar{\sigma}^*\kappa^*_y,$$

$$[\text{ and if } \varphi > 1], \quad c)\quad (1+\beta\mu)(1+a^*) > -\mu\bar{\sigma}^*\kappa^*_y$$

2. *with $a^* < 0$:*

$$a)\quad (1+\beta\mu)(1+a^*) > -\mu\bar{\sigma}^*\kappa^*_y \quad \text{and} \quad b)\quad (1-\beta\mu)(1-a^*) > \mu\bar{\sigma}^*\kappa^*_y.$$

Proof. See Appendix B. ■

To appreciate the implications, consider first the possibility that there is no sovereign-risk channel ($\xi = 0$). In this case the range of parameters for which the equilibrium is determinate is larger if spending is countercyclical ($\varphi < 0$). Interestingly, however, with an endogenous risk

[22] The analytical results in Proposition 1 do not depend on the precise size of the response to inflation, ϕ_π, once the economy has left the lower bound (apart from whether the parameter satisfies the Taylor principle). At first glance this seems to contradict the results in Davig and Leeper (2007). Yet, these authors look at an economy with monetary regime changes in which the Taylor rule satisfies the Taylor principle in one regime but not the other. They show that the equilibrium may remain locally determinate in both regimes in such a setup if the "passive regime" is not too persistent and if—at the same time—in the "active" regime, monetary policy is sufficiently responsive to inflation. This suggests that the monetary response to inflation should figure in the determinacy conditions. Their calculations, however, explicitly exclude the possibility that the passive regime is a lower bound scenario in which monetary policy does not react at all to inflation. Rather they focus on the case in which in both regimes there is some reaction of monetary policy to inflation. If one of the regimes is a lower bound regime, the precise size of the response to inflation in the active regime does not feature in the determinacy considerations.

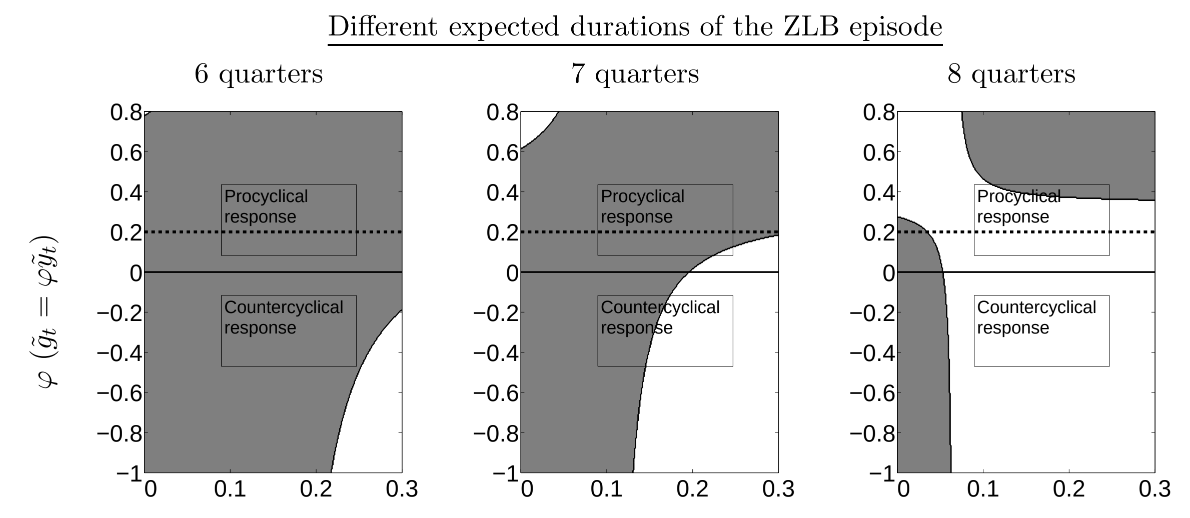
premium, the opposite may hold. More precisely, if $\xi > 0$ and if in addition the conditions of Item 1 of Proposition 2 hold, then subject to some limits on the elasticity of taxes with respect to output, namely, $\phi_{T,y} < 1 - \frac{\kappa\nu}{(1-\beta\mu)\xi}$, the range of fundamental parameters for which the equilibrium is determinate is at least as large with a procyclical spending response, $\varphi \in (0, 1)$, as without any response, and can be larger. Note that this case is the more likely the less elastic the tax revenue responds to the state of the economy (the smaller $\phi_{T,y}$), and the more responsive the interest rate spread to the deficit (the larger ξ). Put differently, a procyclical fiscal stance may reduce the risk of equilibrium indeterminacy (see Appendix B, corollary 6 for details).[23]

This deserves some discussion. While previous work has focused on the case in which sovereign debt and a rising risk premium imply explosive debt, we consider a situation in which debt ultimately will always be stabilized, through one-off tax measures. In such an environment, we find that an economy with an endogenous risk premium can be prone to belief-driven equilibria. At the same time, spending cuts during recessions may actually help to anchor expectations on a unique equilibrium. To see why, assume that during the ZLB period agents expect some drop in output. A drop in output means less tax revenue and, in the absence of a fiscal response, higher deficits, and thus, ultimately, a higher interest rate spread. As this rise in the interest rate spread cannot be offset by monetary action at the ZLB, it immediately raises the real interest rate. As a result, expectations of negative output developments can become self-fulfilling in high-debt economies, with a high and rising interest rate spread weighing heavily on output, thus confirming agents' beliefs in equilibrium. In contrast, a procyclical fiscal stance may be sufficient to prevent an adverse expectational shock from confirming itself, because expected spending cuts would offset the expected decline in tax revenues triggered by a decline in output.

For the baseline parameterization, Figure 3 illustrates the results of Propositions 1 and 2. Each panel of the figure displays results for a different value of μ implying, from left to right, an expected duration of the ZLB episode of 6, 7 and 8 quarters, respectively. For different values of the slope of the risk-premium ξ, measured on the horizontal axis, and the response of government spending to output φ, measured on the vertical axis, we evaluate whether a

[23]Clearly, we here focus on very simple fiscal and monetary rules in order to maintain the analytical tractability. More complicated rules that would make future monetary or fiscal behavior depend on past developments might, in principle, help overcome problems of indeterminacy.

Figure 3: Determinacy regions with endogenous response of government spending



Notes: Determinacy regions with endogenous response of government spending to economic activity during a deep recession. Grey areas mark parameterizations that imply determinacy. y-axis: response of government spending to output, $\varphi\ (\tilde{g}_t = \varphi \tilde{y}_t)$. x-axis: response of the interest rate spread to the deficit, ξ. From left to right: ZLB is expected to bind for 6, 7, or 8 quarters (or, $\mu = 5/6, 6/7, 7/8$).

unique equilibrium exists. Grey areas indicate determinacy regions, while white areas indicate equilibrium indeterminacy. In case the expected duration at the ZLB is long (right panel) and the slope of the risk premium steep, we find that there may be no countercyclical spending policy – of the simple form analyzed here – that ensures determinacy.

On a final note, in this section we have made the risk-premium depend on the expected deficit, as shown in (37). This conforms to the notion that sovereign spreads depend on the expected fiscal position going forward. Quantitatively, however, the results would be very similar if we had, instead, assumed that the sovereign spread depended on the current period's deficit.

3.4 Output effects of spending cuts

In the following we focus on the effects of exogenous spending cuts and limit our analysis to parameterizations for which a stable and unique equilibrium exists. Eventually we are interested in understanding how the effect of cuts in government spending depends on the strength of the sovereign-risk channel. In the simplified model setup that we consider at the moment, the latter is captured by parameter ξ. Since monetary policy may neutralize the effects of the sovereign-risk channel outside the ZLB episode, parameter μ plays a key role in our analysis as well.

For analytical convenience, we analyze the effects of spending cuts during the ZLB episode. In doing so, we follow Woodford (2011) and Christiano et al. (2011) and assume that government spending takes on a value that differs from its steady-state level only while the economy is at the ZLB, namely, a level of $\tilde{g}_t = g_L$. Otherwise government spending is set to its steady-state level.

Proposition 3 *Under the conditions spelled out by Proposition 1 (which ensure that a unique bounded equilibrium exists), let government spending take on a value of g_L whenever the lower bound is binding, and 0 otherwise. As before, define $a = \mu + \mu\xi\phi_{T,y}\bar{\sigma}$, and $b = \mu + \mu\bar{\sigma}\xi$. Then, while the economy is at the ZLB, output is given by*

$$y_L = \vartheta_r(\log(1+i^d) - \Gamma) + \vartheta_g\, g_L,$$

where

$$\vartheta_r = \frac{\bar{\sigma}(1-\beta\mu)}{(1-\beta\mu)(1-a) - \mu\bar{\sigma}\kappa_y} > 0 \tag{39}$$

and

$$\vartheta_g = \frac{(1-\beta\mu)(1-b) - \mu\bar{\sigma}\kappa_g}{(1-\beta\mu)(1-a) - \mu\bar{\sigma}\kappa_y}. \tag{40}$$

Proof. See Appendix B. ■

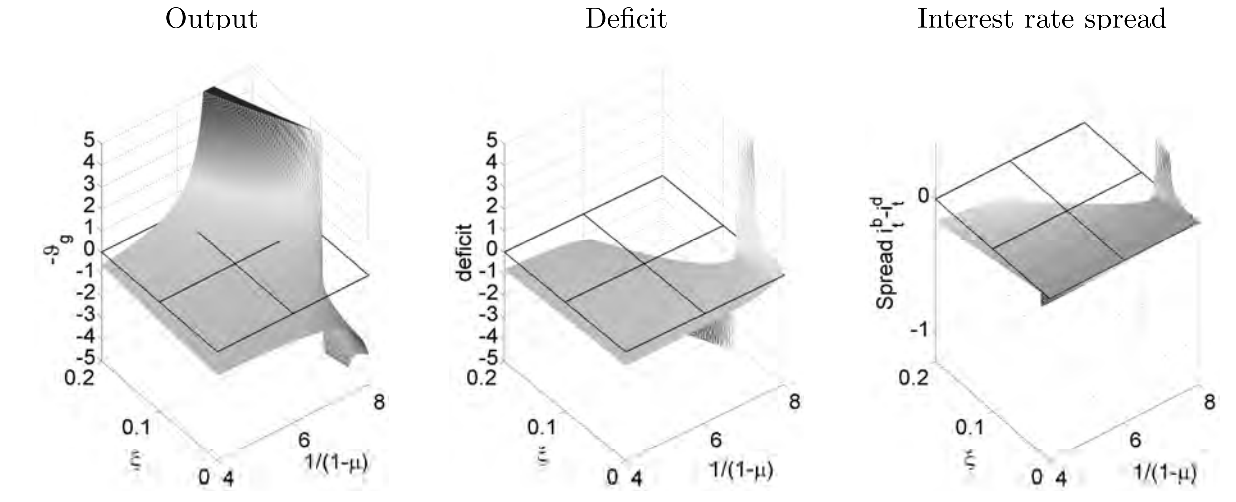
Note that ϑ_g provides a measure for the government spending multiplier on output at the ZLB. It is characterized in more detail by Corollary 7 in Appendix B.5. Specifically, under the determinacy conditions established above, equation (40) implies that the multiplier is positive if and only if

$$(1-\mu) - \frac{\mu\bar{\sigma}\kappa_g}{1-\beta\mu} > \mu\xi\bar{\sigma}. \tag{41}$$

If this condition is satisfied, a spending *cut* at the ZLB will reduce output. If $\xi = 0$, this will always be the case; moreover, the government spending multiplier will be strictly larger than one; see Christiano et al. (2011) and Woodford (2011). In contrast, if $\xi > 0$, the government spending multiplier at the ZLB may actually be negative, such that spending cuts raise output.

The left panel of Figure 4 illustrates this result graphically. It displays the output effect of a government spending cut during the ZLB episode for different levels of fiscal strain, measured

Figure 4: Effects of early retrenchment



Notes: The figure shows the effects of a unit cut in government spending for the length of the ZLB episode. Effect on output (left panel), on the deficit (center: negative means deficit falls) and on the interest rate spread (right). On the axes: responsiveness of interest spread to expected deficit, ξ, and expected duration of ZLB episode: $1/(1-\mu)$. Only parameterizations that imply determinacy are shown. For better readability, multipliers and deficits were capped at the maximum level indicated in the charts.

by alternative values for ξ, and for different assumptions regarding the expected length of the recession, as measured by alternative values of $1/(1-\mu)$. The other parameters underlying these computations remain as laid out earlier. In the same figure we also show the response of the budget deficit (middle panel) and the interest rate spread (right panel)

Output's response to a spending cut depends on both dimensions under consideration. Consider first the case where $\xi = 0$, that is, a situation when there is no sovereign-risk channel. In this case, a spending cut induces a sizeable decline of output. In fact, for an expected duration of the ZLB episode of eight quarters, the government spending multiplier on output reaches a value of about 3, a result recently stressed in Christiano et al. (2011). The underlying mechanism is well understood: the deflationary effect of spending cuts cannot be accommodated by a reduction in policy rates and thus triggers an increase in the real interest rate, which crowds out private demand. The effect is stronger, the longer the expected duration of the ZLB, as private demand is determined by the expected path of current *and future* short-term real interest rates.

Turning to the sovereign-risk channel, we focus first on the case where the ZLB is expected to be short, such that monetary policy is expected to be constrained for four quarters only. In this case, as the interest rate spread becomes more responsive to the deficit, that is as ξ takes on bigger values, the multiplier tends to decline, that is, output tends to fall by less. Yet the role of the sovereign-risk channel is clearly limited, even for very high values of ξ. This is due to the fact that monetary policy is expected to be able to offset the effect of sovereign risk on private interest rates in the near future.

However, if monetary policy is expected to be constrained for an extended period, the sovereign-risk channel has a strong bearing on the fiscal transmission mechanism. In fact, if ξ and μ both take on high values, the sign of the output multiplier changes. A spending cut during the ZLB episode then is expansionary. To understand this finding, it is useful to consider the response of the deficit and of the risk premium. Note that for most parameterizations, a cut in government spending reduces the deficit (see also Erceg and Lindé (2010*b*)). If fiscal strain is pervasive, this leads to a considerable decline in the risk premium. This, in turn, reduces the interest rate spread, stimulates private demand and tax revenues—setting in motion a virtuous cycle of a further decline in interest rate spreads, increased economic activity and a further improvement of the fiscal outlook.

In sum, we find in our simplified model setup a possibly important role for the sovereign-risk channel. In fact, fiscal retrenchment may be expansionary in the presence of severe fiscal strain, provided that monetary policy is severely constrained and cannot cushion the adverse effects of sovereign risk on private-sector borrowing conditions. In this case, a cut in government spending that reduces the deficit may set in motion a virtuous circle, which brings down interest rate spreads and stimulates economic activity.[24]

4 Dynamic analysis

We now turn to a numerical analysis of the full model, as outlined in Section 2 above. This allows us to revisit our analytical results while accounting for the possibility that sovereign

[24]The literature has emphasized that retrenchment after the ZLB episode can stimulate economic activity while the economy is still at the ZLB, for example, Corsetti, Kuester, Meier and Müller (2010) and Woodford (2011), provided that the future retrenchment is persistent enough. For completeness, Appendix A discusses how the sovereign-risk channel affects the conclusions for such a timing of retrenchment. We find that, for the packages we consider, there are parameterizations under which such a future retrenchment increases output at the ZLB in the presence of a sovereign-risk channel, while it crowds out output in its absence. The opposite is not true. In particular, whenever a future retrenchment package of the form we consider crowds in output at the ZLB in the absence of a sovereign-risk channel, it also crowds in output at the ZLB in the presence of the sovereign-risk channel. Indeed, any positive crowding in effect of a future retrenchment is even stronger if the sovereign-risk channel is active.

risk depends on the expected debt level rather than the expected deficit as in Section 3. In order to highlight the role of monetary policy, we focus again on a ZLB scenario. However, we depart from the simplifying assumption that the expected duration of the ZLB episode is constant. Instead, we envisage a scenario in which a) the initial debt level matters for the depth of the recession and b) in which fiscal retrenchment may alter the length of the lower bound episode.[25]

4.1 Deep recessions, sovereign risk, and fiscal retrenchment

To set the stage for our analysis, we subject the model economy to a large recessionary shock that pushes the economy to the ZLB. We specify a first-order autoregressive process for $\widehat{e}_t$ in order to capture in a stylized manner the output loss in the U.S. during the 2007–2009 recession. In particular, the Congressional Budget Office (2011) estimates that the output gap reached 6.7 percent in 2009 and that it will still be at a 1.7 percent level in 2014 and at 0.5 percent in 2015.[26] For the simulations, we also assume that taxes do stabilize the debt level, but only very gradually.[27] While the simulations for the baseline scenario assume that taxes are raised in a lump-sum manner, Section 4.2 assesses the extent to which distortionary taxation would affect our conclusions.

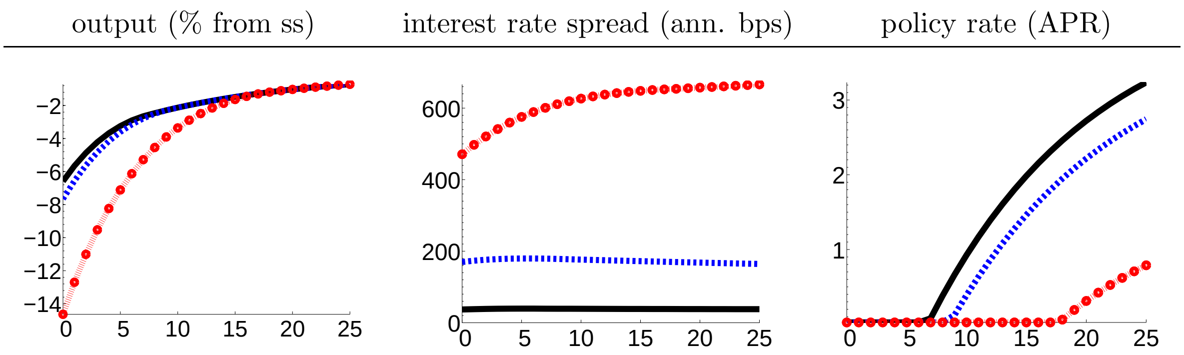
Figure 5 shows the evolution of output, the interest rate spread and the policy rate in response to the recessionary shock. It displays the behavior of the economy in the absence of any discretionary fiscal policy measure for three different initial levels of government debt: 60 percent of GDP (black solid line), 90 (blue dashed line) and 115 percent (red dots). The adjustment dynamics differ substantially in that the decline in output and the rise of the interest rate spread are stronger if initial debt is high. To understand this result, note that the shock induces an increase in the budget deficit, which leads to a build-up of public debt and thus an increase in the sovereign-risk premium. Moreover, since the shock pushes the

[25] We solve the model economy under perfect foresight using standard techniques.

[26] Given the process $\log(e_t) = \rho_e \log(e_{t-1}) + u_t$, we set $u_0 = -0.1525$ (setting $u_t = 0$ for all other periods) and $\rho_e = 0.93$ to roughly replicate those values for our baseline economy with a level of debt-to-GDP of 60 percent. At the time of writing, the CBO had not yet published output gap estimates and forecasts based on the revision to the national income and product accounts released by the BEA on July 29, 2011.

[27] In particular, unless noted otherwise, we set the response parameter $\phi_{T,b^g} = 0.014$ for the first 30 quarters. This response is twice as large as would be required to ensure stable debt dynamics in the absence of adverse movements in the risk premium. At the same time, the adjustment of taxes is slow enough that absent government spending cuts, the debt burden will be reduced only in a very gradual manner. Beyond quarter 30, ϕ_{T,b^g} rises to twice the previous value, ensuring that the sovereign debt will eventually be stabilized even for the higher levels of sovereign indebtedness (and the correspondingly high risk premium).

Figure 5: Deep recession with different initial levels of sovereign debt



Notes: The figure shows responses to the recessionary shock for different initial debt levels. Black solid line: 60 percent debt-to-GDP, blue dashed: 90 percent, red dots: 115 percent. Output is expressed in terms of percentage deviations from the steady state, the risk premium in annualized basis points, and the interest rate in annualized percentage rates.

economy to the ZLB, monetary policy is unable to offset the spillover from sovereign-risk to private interest rates. Private expenditure thus falls further, adding to the initial decline in output. This effect is the stronger the higher the initial debt level, because the relationship between public debt and sovereign risk is fundamentally nonlinear.

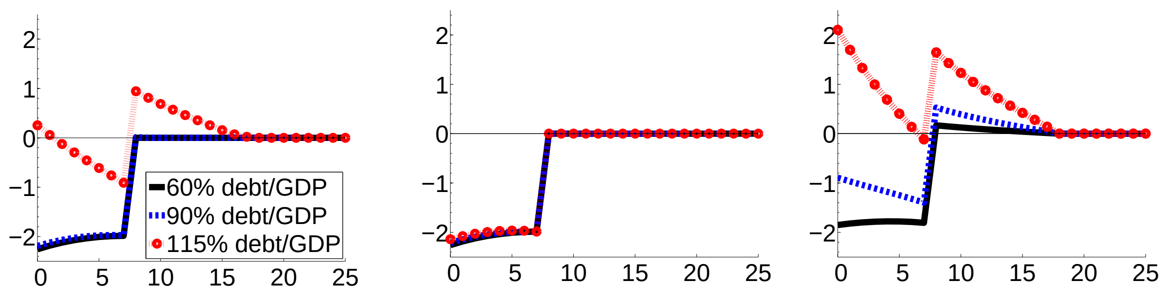
In our simulations, a persistently high risk premium also implies that the time span over which the lower bound remains binding is longer. This becomes apparent once one considers the dynamics of the policy rate shown in the right panel of Figure 5: the ZLB episode is extended by as much as 11 quarters if sovereign debt is high. Hence, our dynamic analysis delivers a first additional insight: not only does the importance of the sovereign-risk channel depend on whether the central bank is constrained in setting the interest rate to the desired level, but sovereign risk may itself be an important determinant for how strongly monetary policy is constrained in the face of certain shocks.

We now analyze the effects of government spending cuts that are assumed to take place in the recessionary environment just described. Specifically, in order to mimic the setup of Section 3, we consider a sequence of spending cuts that last for two years and start at the onset of the recession, that is, we consider an “immediate retrenchment” scenario. We discuss alternative timing assumptions in Section 4.2. The spending cuts equal 2 percent of (steady-state) GDP per period. Figure 6 shows their effect relative to the baseline scenario. The panels on the left

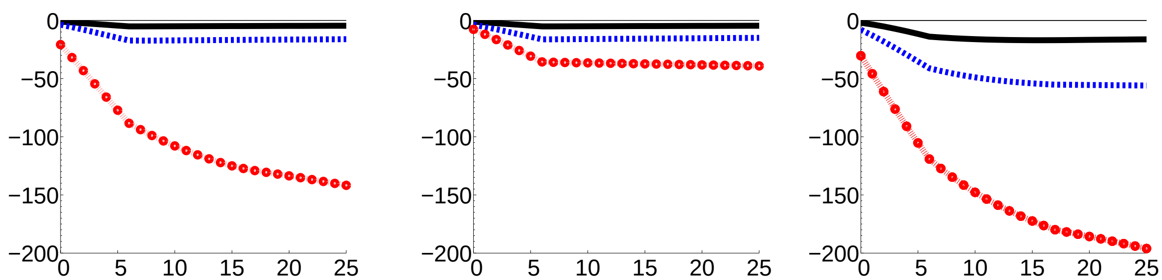
Figure 6: The effect – relative to the baseline – of immediate retrenchment

Timing of ZLB exit endogenous	ZLB lasts for 7 quarters	ZLB lasts for 18 quarters

a) Effect on output (% deviation from ss)



b) Effect on risk premium (bps, annualized)



Notes: The effect of an immediate retrenchment in government spending by 2 percent of steady-state output for 8 quarters on output (top) and the risk premium (bottom). Solid black line: 60% initial debt to GDP ratio, dashed blue line 90%, dotted red line: 115%. Left panel: the effect of the retrenchment package when the timing of the exit from the ZLB is endogenously determined according to equation (36). Other panels: for each initial debt level, the depth of the recession without retrenchment is calibrated such that it implies that the ZLB will bind until quarter 6 (center) and 17 (right). For the center and rightmost panels, regardless of the austerity package implemented, monetary policy is assumed to keep the nominal rate at zero for that time period (until quarter 6 and 17, respectively).

show the response of output (top) and the interest rate spread (bottom). In order to isolate the effect of a binding ZLB, the panels in the middle and on the right show the response of the same variables under the assumption that the exit from the ZLB is not endogenous, but fixed exogenously at 7 and 18 quarters. These lengths correspond, respectively, to the duration of the lower bound episode with a 60 and 115 percent debt-to-GDP ratio in Figure 5.[28]

We find, first, that for all initial debt levels, the retrenchment package is effective in reducing the deficit (not shown). Thus the level of sovereign debt and thereby the risk premium (in line also with Figure 4) decline. In addition, monetary policy does leave the lower bound somewhat earlier for all debt levels (not shown). However, the spending cuts have quite different effects on output in the different scenarios we consider in our simulations. If initial debt is low, in the wake of the spending cuts output falls initially by more than 2 percent, as private expenditure declines. Yet, if initial debt is high, the initial output response is actually positive, reflecting a strong increase in private expenditure. To understand this finding, recall that spending cuts affect real interest rates through two channels. On the one hand, the deflationary effect of spending cuts raises, all else equal, real interest rates as it cannot be met by a reduction in policy rates at the ZLB. On the other hand, the reduction of public debt reduces sovereign risk, thereby lowering private interest rate spreads at the ZLB. The strength of the effect operating through the second channel increases in the initial level of debt, as, again, the relationship between public debt and sovereign risk is fundamentally nonlinear. In our simulations it dominates the final outcome if we assume an initial debt level of 115 percent of GDP.

As stressed above, the impact of the recessionary shock and the period of time for which monetary policy is constrained depend on the initial level of debt. At the same time, the initial level of debt determines the quantitative importance of the sovereign-risk channel. To isolate the latter dimension, the panels in the center and right columns of Figure 6 show the effect of the austerity packages on output for different debt levels, but fixing the length of the ZLB episode exogenously. If the lower bound is expected to be binding only for a short period of time (center panels), the effects of spending cuts hardly vary with the debt level. The importance of sovereign risk thus appears limited. However, in line with our results in

[28]In the underlying computations, for each debt level, we rescale the initial recessionary shock such that the ZLB binds for the desired length of time.

Section 3, this changes quite dramatically if the lower bound is expected to be binding for a longer time and if, therefore, monetary policy cannot sterilize the effect of the risk premium on economic activity for an extended period of time (right panels). In this case, we find that spending cuts have a positive and lasting effect on economic activity if the initial level of debt is high enough and therefore the risk premium is high (dotted red line, right panel). Moreover, the output costs of spending cuts fall notably also for the intermediate, 90 percent, level of debt to GDP (dashed blue line in the rightmost panel). Overall, we thus find our earlier results confirmed: fiscal strain may alter the fiscal transmission mechanism through the sovereign-risk channel, provided that monetary policy is expected to be constrained for an extended period.

4.2 Further considerations

This section assesses the sensitivity of the previous results with respect to modifications that may have a bearing on the workings of the sovereign-risk channel. We start by assessing how an alternative timing of cuts affects the outcome of fiscal retrenchment. We then explore alternative assumptions on how taxes are raised and monetary policy is conducted.

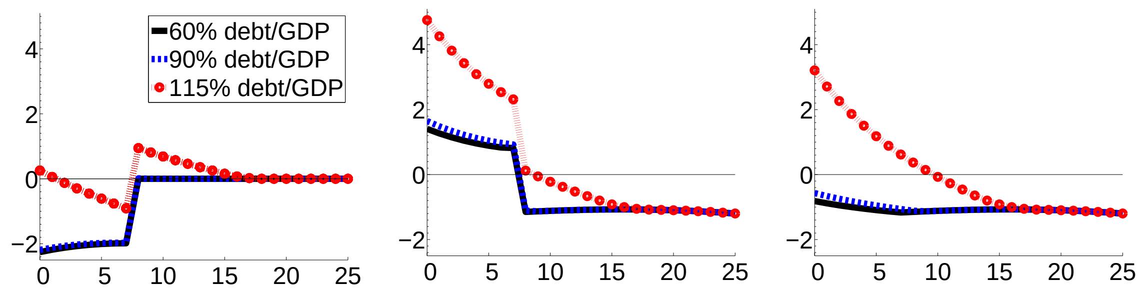
4.2.1 The timing of spending cuts

So far, we have focused on spending cuts that take place immediately, that is, at the time the recessionary shocks impact the economy. Such a scenario is conceptually close to the one considered in Section 3, which we adopted for analytical convenience. We now assess in more detail the role of the timing of retrenchment measures on the basis of model simulations. Specifically, while the left panels of Figure 7 repeat the results for an immediate retrenchment that were reported in the left column in Figure 6, the remaining columns report the impact of two alternatively timed consolidation packages. The center panels show the response of the economy to a package of spending cuts of 2 percent of steady-state GDP that starts two years after the initial recessionary impact and that lasts for 10 years (a "medium-term retrenchment"). The right panels show results for a combination of the previous two packages (a "persistent retrenchment," so spending cuts start immediately and last for 12 years in total). As before, the figures report the effect of these different packages depending on the initial level of debt (the range of the axes has been rescaled to accommodate the range implied by these simulations).

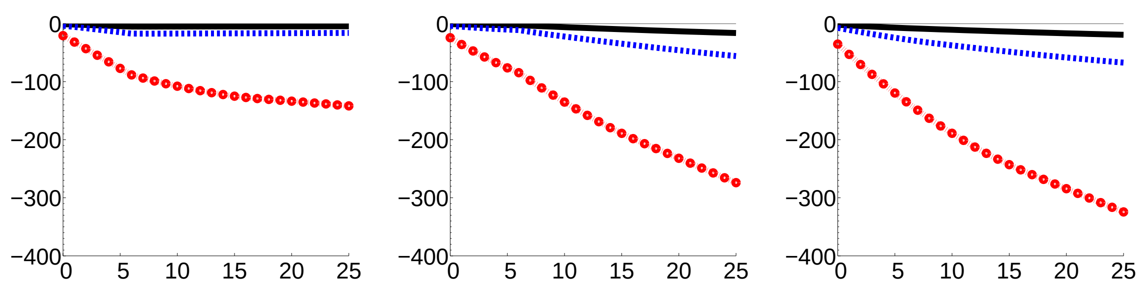
Figure 7: The timing of retrenchment

Immediate retrenchment	Medium-term retrenchment	Persistent retrenchment

a) Effect on output (% deviation from ss)



b) Effect on interest rate spread (bps, annualized)



Notes: The effect of an immediate retrenchment in government spending by 2 percent of steady-state output for 8 quarters on output (top) and the risk premium (bottom). Solid black line: 60% initial debt-to-GDP ratio, dashed blue line 90%, dotted red line: 115%. For all the panels the timing of the exit from the ZLB is endogenously determined according to equation (36). Left panels: immediate retrenchment (defined in the text), center: medium-term retrenchment, right: persistent retrenchment.

Under medium-term retrenchment (center panels), output gains from fiscal forward guidance are possible in line with the results in Corsetti, Kuester, Meier and Müller (2010). In this case most or all of the spending cuts are implemented when monetary policy can again add to stabilizing the economy by means of lowering the policy rate. Spending cuts reduce demand and inflation at the time of the cuts, which leads the central bank to reduce the real interest rate. The prospect of a lower long-term real rate crowds in consumption in the early periods of the recession, and thus output, and quite strongly so. This leads to higher tax revenues and implies an immediate reduction in the risk premium, which further stimulates demand through reduced interest rate spreads. Note that such medium-term retrenchment is the more stimulative the weaker the fiscal situation is (dotted red line vs dashed blue line in the central panels). Last, the persistent retrenchment scenario is a combination of the two previous scenarios. In the simulations shown in the right column of Figure 7, persistent retrenchment has the strongest effect on the risk premium (red dotted line), but its output effects are generally somewhat smaller than in the medium-term retrenchment scenario.[29]

4.2.2 Distortionary taxation

In the absence of a binding ZLB, higher distortionary taxes reduce economic activity. To the extent that an early retrenchment reduces the need for distortionary taxation in the future, such an early retrenchment would be expected to be less harmful to economic activity than we have reported so far. Figure 8 assesses this possibility, again by reporting results for three debt levels.[30]

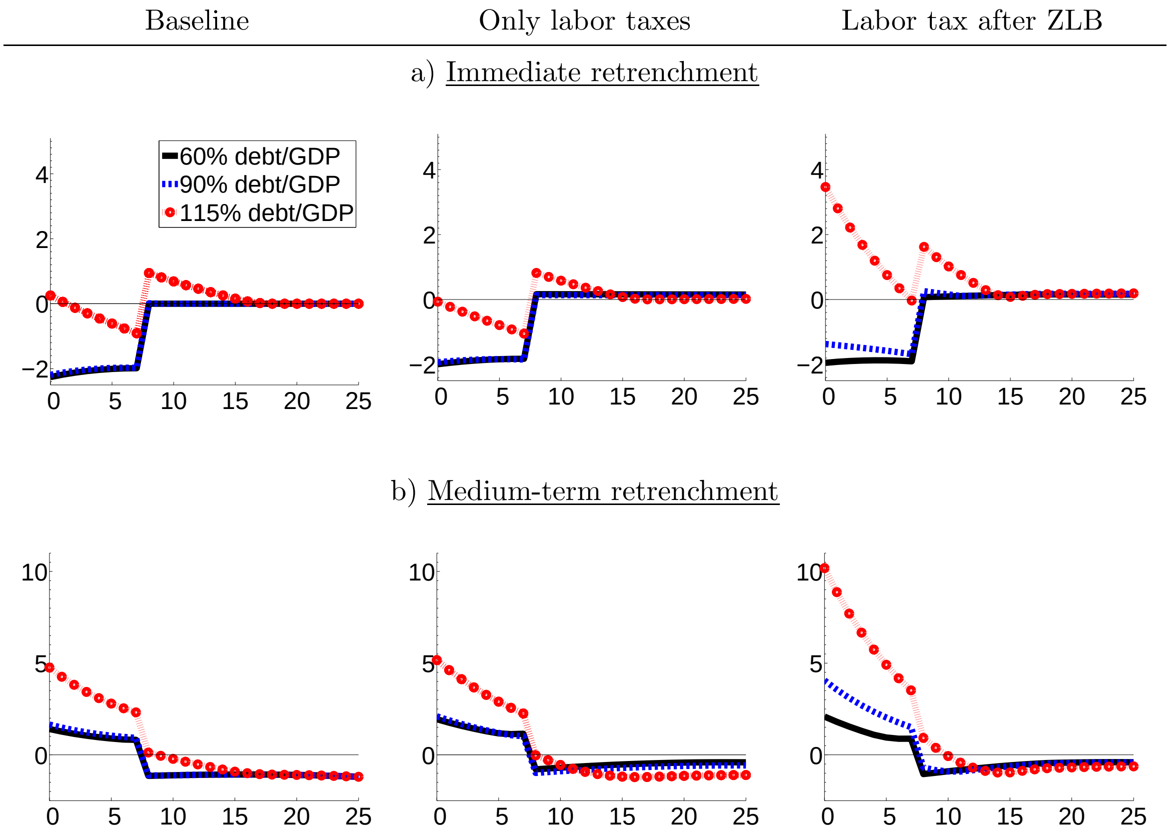
The first column repeats the responses in the baseline, in which only lump-sum taxes are used. The second column shows how distortionary taxation alters the effect of the spending cuts. In the simulations, distortionary taxation, if used throughout, has little bearing on how retrenchment affects output. While lower future distortionary taxation stimulates future economic activity, and thus current demand, in our simulations, a second, countervailing effect is present. Without a retrenchment, distortionary tax rates rise more strongly (with a view toward stabilizing the rising debt burden). As stressed by Eggertsson (2011), to the extent that higher labor tax rates raise inflation, they may actually help to stabilize output if the economy is stuck at the ZLB. Instead, an early retrenchment reduces the labor tax burden and exerts a negative effect on activity, which, in the simulations shown here, outweighs the positive effects due to the reduced tax burden in later periods.

One may argue, however, about the extent to which taxes would actually be increased while the economy remains in a lower bound situation. Since the timing of taxation is important for the aforementioned results, the rightmost column of Figure 8 shows results for a different

[29] For the higher, 115 percent level of debt, the persistent retrenchment continues to crowd in economic activity for most of the time that the economy remains constrained by the lower bound on interest rates. It crowds in initial output by less than the medium-term retrenchment alone, because, in the simulations shown, already the medium-term retrenchment does considerably reduce the sovereign debt burden relative to the baseline. Any additional retrenchment yields relatively little reduction in the sovereign-risk premium.

[30] For this exercise, the labor tax rate is calibrated to 35 percent in the steady state, in line with the 2006 average U.S. marginal income tax rate reported by Barro and Redlick (2011). Fluctuations in labor taxes ensure that (28) holds. For the scenarios with labor taxes, we set $\phi_{T,b^g} = 0.0193$ for all periods.

Figure 8: Effects of retrenchment on output depending on taxation



Notes: The effect of retrenchment on output spending for different assumptions about taxation. The left column repeats the effect of retrenchment with lump-sum taxes (as in the baseline). The center panel shows the effect of retrenchment when labor taxes (instead of lump-sum taxes) are used throughout. The right column shows the effect of retrenchment if for the first 18 quarters taxes are lump-sum but distortionary thereafter. For the simulations in which labor taxes are used we have rescaled the initial shock such that for each debt level the size of the recession and the length of the ZLB episode without retrenchment is roughly comparable to the baseline responses shown in Figure 5.

taxation scenario. It aims to purge the effects of retrenchment from the effects that labor taxes have in a lower bound situation. In particular, the responses in that column are based on the assumption that for the first 18 quarters lump-sum taxes are in place that prevent the debt from exploding. Only thereafter do labor taxes (and no longer any lump-sum taxes) ensure that the debt is stabilized. That is, labor taxes respond only well after the lower bound has ceased to bind. For the 60 percent debt level, the difference to the case of lump-sum taxation is small and the effects are very similar to the baseline. For the 90 percent debt level, the output effects of retrenchment turn more favorable. However, this mostly affects the medium-term retrenchment packages. Finally, results for the highest debt level shown here, 115 percent debt, illustrate that the sovereign-risk channel may be stronger in the case with distortionary taxation than without, provided that changes in distortionary tax rates materialize only after the the ZLB has ceased to bind. Intuitively, for this level of sovereign indebtedness, the heightened risk premium demands relatively strong increases in distortionary taxes to finance the debt burden, which by itself worsens the recession. As a result, an early retrenchment now crowds in output throughout, and much more strongly so than in the case with lump-sum taxes only.

4.2.3 Degree of absorption of risk premium

Next, we consider the sensitivity of our results with regard to the extent to which the central bank will, in fact, be capable of sterilizing the effect of sovereign risk on private borrowing rates. In particular, our baseline simulations have assumed that once out of the lower bound situation, the central bank can set interest rates in a way that prevents the sovereign risk from affecting economic activity.

We now assess, instead, the effect of spending cuts if the central bank does not, or cannot, fully neutralize the sovereign spread. The lower panels of Figure 9 show the results of fiscal retrenchment when the central bank does not (or cannot) respond as vigorously to the interest rate spread. In particular, we assume that the central bank's response is only three-quarters the size of the value of ϕ_ω that we used in the previous simulations. Namely, we set $\phi_\omega = 0.75(\pi_b + s_\Omega)$. Relative to the case of full sterilization (upper panel), we find, in particular, that the effects of early retrenchment are less detrimental to economic activity if the initial level of debt is high, and, hence the sovereign-risk channel potentially important. However, for higher debt levels the outcomes of retrenchment are generally more favorable even for immediate debt levels of, say, 90 percent to GDP if the central bank cannot perfectly sterilize the sovereign risk premium in the future.

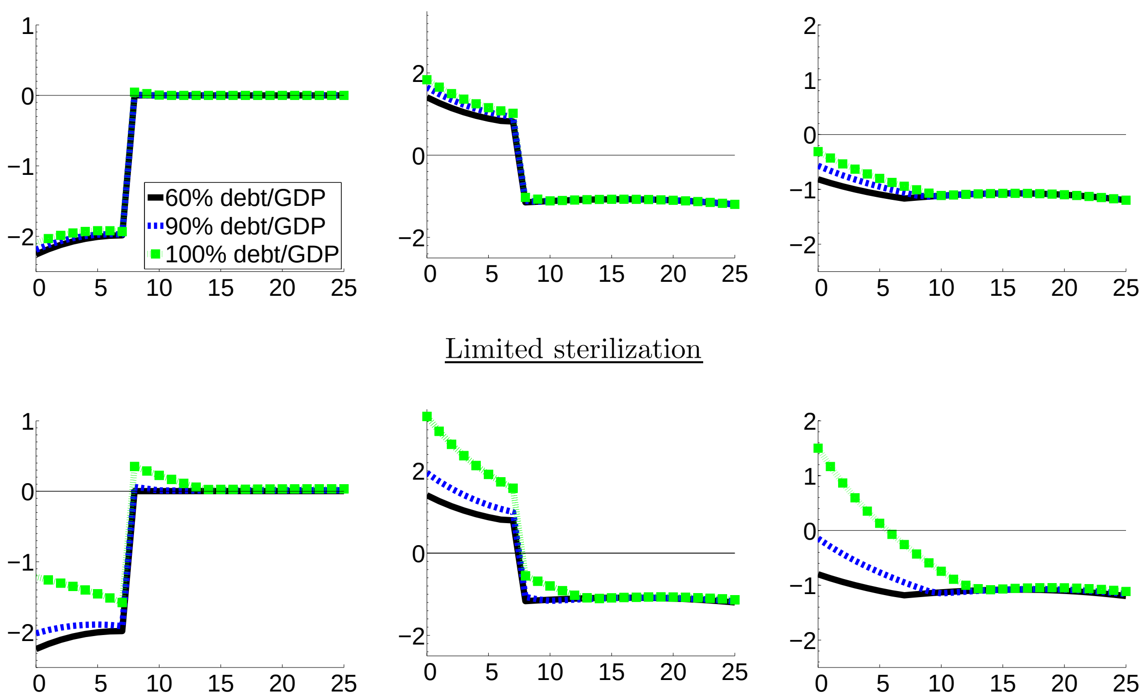
4.2.4 Variations in the spillover or risk premium

We have so far treated the relationship of the interest spread and the level of debt as constant over time. Yet, it is well known that there are shifts in attitudes toward risk, and thus in the relationship between the debt level and the sovereign risk premium as well. Figure 10

Figure 9: Effects of retrenchment on output when CB cannot fully neutralize risk premium

Immediate	Medium-term	Persistent

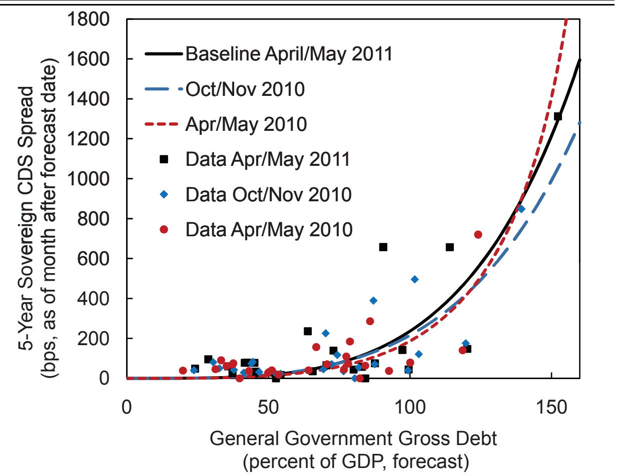
Baseline – full sterilization



Notes: The effect of the three retrenchment packages (described in the notes to Figure 7), when the central bank does not fully absorb the risk premium even once the economy has left the lower bound. Top row shows the baseline responses for 60, 90, and 100 percent initial debt-to-GDP levels. Bottom row shows the case when the response coefficient ϕ_ω is 3/4 of the size of the benchmark calibration.

highlights this for three different dates of the World Economic Outlook forecasts: April 2010, October 2010, and April 2011. The CDS spreads are taken at the beginning of the month following publication of the forecast.

Figure 10: Sovereign risk premium at different points in time



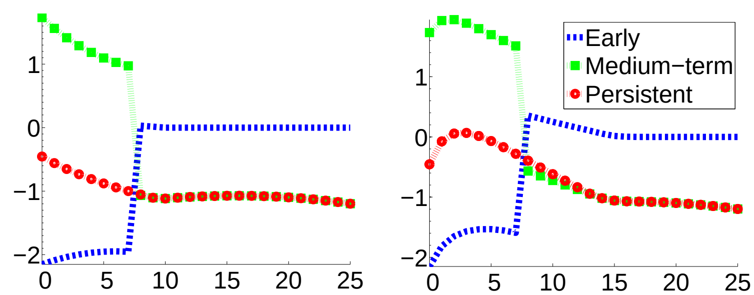
Notes: Same as Figure 2, but for different times. The figure shows 5-year sovereign CDS spreads for industrialized countries against forecasts for gross general government debt over GDP. The debt forecasts are IMF World Economic Outlook forecasts as of April 2011, October 2010 and April 2010. The CDS spreads are taken on the first day of the following month. For the April forecasts, the debt/GDP ratio forecast is for the full year. For the October forecast, it is for the following year. The lines shown are curves fitted to the risk premium

Similarly, Figure 1 suggests that the relationship between public debt and the private risk premium may be somewhat uncertain. In the context of our model, we proxy for such an uncertainty by allowing for a random increase in the spillover from the sovereign risk premium to the interest rate spread. Specifically, we assume that initially the pass-through from sovereign risk to private-sector risk is 0.3, in line with the correlation for low-spread economies in Figure 1, but that in each period there is a 20 percent probability that the spillover increases to the correlation for high-spread economies in the latter figure – and will stay at that level thereafter.

Figure 11 shows the results for a debt level of 110 percent (under the calibration on data for April 2011). These results suggest, and perhaps not surprisingly, that the importance of the sovereign-risk channel increases under these circumstances.

Figure 11: Change of spillover – 110 percent debt

no risk of higher spillover	risk of higher spillover



Notes: The effect of the three retrenchment packages (described earlier in the text) when the degree of spillover can shift. The sovereign risk premium curve is taken in April 2011. Left: spillover permanently stays at $\alpha_\psi = 0.3$. Right: a 20 percent chance per period that the spillover shifts to $\alpha_\psi = 0.7$. Shown is the average effect of retrenchment.

5 Conclusion

Most industrialized countries are facing a period of significant fiscal consolidation, including sizeable spending cuts. How much will these cuts hurt economic activity? While standard multiplier analysis suggests significant headwinds for growth, the current paper shows that the effects of fiscal retrenchment will depend on the precise circumstances under which they are enacted.

In this paper, we consider two conditions that appear to characterize well the current macroeconomic stance in a number of OECD countries, namely, i) severe fiscal strain, as evidenced by a high sovereign risk premium; and ii) a limited capacity of monetary policy to reduce policy rates, given the zero lower bound. We formally analyze the role of the sovereign risk channel within a variant of the model proposed by Cúrdia and Woodford (2009). Two sets of results stand out. First, sovereign risk increases the indeterminacy problem for constrained monetary policy. In particular, private-sector beliefs about a weakening economy can become self-fulfilling, driving up risk premiums and choking off private demand. In this environment, an expected pro-cyclical fiscal stance, that is, tighter fiscal policy, can help to ensure determinacy.

Second, we use a simplified version of our model to show analytically that the sovereign risk channel may, in principle, alter the sign of the output multiplier of government spending, as upfront fiscal tightening leads to lower funding costs throughout the economy. Simulations of the full-fledged model confirm this finding. Indeed, a recessionary shock may simultaneously restrict monetary policy and cause fiscal strain, especially where public debt is already high at the outset. Under these circumstances fiscal retrenchment is likely to be less detrimental to economic activity or may have beneficial effects relative to a scenario without sovereign risk.

In closing, we emphasize three caveats. First, both fiscal strain and the constraints on monetary policy may need to be quite severe in order for government spending cuts to actually stimulate economic activity. Second, in our simulations, a fiscal retrenchment is no miracle cure for the economy's ills. In particular, in all our simulations the recession remains deep even if the fiscal retrenchment stimulates economic activity relative to an even bleaker baseline. Third and last, here we have focused on fiscal multipliers under a "go-it-alone" policy that does not involve an outright bailout or temporary financial support at below-market rates from international institutions, both of which would somewhat weaken the case for an expansionary fiscal retrenchment.

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A Retrenchment after the ZLB phase

The analytical results presented in Section 3 pertain to fiscal retrenchment while the economy is still at the ZLB. For completeness, we also consider in this appendix a retrenchment that is designed to take effect only once the economy has left the ZLB. As discussed in Corsetti, Kuester, Meier and Müller (2010), fiscal consolidation some time in the future does reduce demand and inflation contemporaneously but may have positive output effects well before it is implemented. In fact, if spending consolidation is implemented when the central bank is no longer constrained by the ZLB, in reaction to its effect on inflation the policy rate (R_t) will fall in both nominal and real terms (recall, $\phi_\pi > 1$). In anticipation of a path of lower interest rates, long-term interest rates will contract as of today, thereby crowding in consumption and output even if the economy continues to be at the ZLB. However, by the very nature of this transmission mechanism, the exact timing of consolidation is crucial. When firms anticipate the future drop in demand, because of nominal rigidities they start to reduce prices before government demand actually falls. In other words, inflation falls in anticipation of the retrenchment. If much of the retrenchment happens too close to the period in which the economy has left the ZLB, its effect on the real rate of interest can be perverse, that is, real rates may rise while the economy is still at the ZLB—hampering the recovery. In order to assess the effects on economic activity more formally, we state the following proposition and corollary.

Proposition 4 *In the economy specified in Proposition 1, let $\tilde{g}_t$ take on a value of $= 0$ whenever the ZLB is binding. Once the ZLB ceases to bind, $\tilde{g}_t = g_a < 0$, in the first period,* *and subsequently with probability $\varrho \in [0,1)$. Otherwise $\tilde{g}_t = 0$ forever. Assuming that the conditions for determinacy are satisfied, output while at the ZLB is given by*

$$\begin{aligned} y_l \quad = \quad \tfrac{1}{d} \Big[\quad & \bar{\sigma}(1-\beta\mu)[\log(R)-\Gamma] + (1-\mu)(1-\beta\mu)(1+\bar{\sigma}\xi\phi_{T,y})(y_a - g_a) + \bar{\sigma}(1-\mu)\pi_a \\ & - (1-\mu)(1-\beta\mu)\bar{\sigma}\xi(1-\phi_{T,y})g_a \quad \Big], \end{aligned} \tag{42}$$

where $d = (1-\beta\mu)[1-a] - \bar{\sigma}\mu\kappa_y$, $a := \mu + \mu\xi\phi_{T,y}\bar{\sigma}$ as in Proposition 1, and y_a and π_a denote, respectively, output and inflation in the austerity period, equal to

$$y_a = \frac{(1-\varrho)(1-\beta\varrho) + \bar{\sigma}(\phi_\pi - \varrho)\kappa_g}{(1-\varrho)(1-\beta\varrho) + \bar{\sigma}(\phi_\pi - \varrho)\kappa_y} g_a, \tag{43}$$

and

$$\pi_a = \frac{(1-\varrho)(\kappa_y - \kappa_g)}{(1-\varrho)(1-\beta\varrho) + \bar{\sigma}(\phi_\pi - \varrho)\kappa_y} g_a. \tag{44}$$

Proof. See Appendix B. ■

Corollary 5 *Under the conditions in Proposition 4,*

1. *if $\xi = 0$, retrenchment after the ZLB phase enhances economic activity at the ZLB phase unless too much of it is expected to occur too close to the exit from the ZLB. More precisely, there exists a value of $\varrho \in [0,1)$ such that $y_l > 0$ if $g_a < 0$. This is the case for any $\varrho > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.*
2. *Provided that the effect of future austerity on y_l is positive, the magnitude of such an effect will be larger the more sensitive the economy is to the risk premium (the larger is ξ).*
3. *In addition, there are parameterizations for which the effect of future austerity is positive if $\xi > 0$, while it is negative if $\xi = 0$.*

Proof. See Appendix B. ■

Here we are primarily interested in the output effects during the ZLB period. Corollary 5 provides a detailed characterization. In the absence of a risk-premium channel, the corollary shows that future spending cuts that take effect after the end of the ZLB phase raise output in the ZLB phase unless too much of the retrenchment is expected to occur too close after the exit from the ZLB, i.e., future retrenchment efforts need to be sufficiently persistent; compare Corsetti, Kuester, Meier and Müller (2010) and Woodford (2011). The corollary also highlights that the risk-premium channel would enhance any crowding in of output while at the lower bound; see Item 2 of Corollary 5). In addition, for certain parameterizations, future retrenchment, even if not particularly persistent, may stimulate output at the ZLB if the fiscal situation is weak ($\xi > 0$) while such a retrenchment would have negative effects in the absence of the sovereign risk channel, that is, for $\xi = 0$ (Item 3).

B Proofs

B.1 Proof of Proposition 1

The economy, stripped from exogenous variables, is given by

$$E_t z_{t+1} = A z_t,$$

where $z_t = [\tilde{y}_t; \widehat{\pi}_t]$ and

$$A = \frac{1}{a\mu\beta} \begin{bmatrix} \mu\beta + \bar{\sigma}\mu\kappa_y & -\bar{\sigma}\mu \\ -a\kappa_y & a \end{bmatrix},$$

where $a = (\mu + \mu\bar{\sigma}\xi\phi_{T,y})$. The Blanchard-Kahn conditions for determinacy require that matrix A have two roots outside the unit circle. Woodford (2003) gives the following necessary and sufficient conditions for determinacy:
either (Case I): (i) $det(A) > 1$, (ii) $det(A) - tr(A) > -1$, and (iii) $det(A) + tr(A) > -1$,
or (Case II): (i) $det(A) - tr(A) < -1$ and (ii) $det(A) + tr(A) < -1$.

In the current case, $det(A) = \frac{1}{a\mu\beta}$ and $tr(A) = \frac{1}{a\mu\beta}[\mu\beta + \bar{\sigma}\mu\kappa_y + a]$. Since both $det(A) > 0$ and $tr(A) > 0$ Case II cannot be satisfied. Checking Case I, condition (iii) holds since both terms are positive. Condition (i) is equivalent to condition a) in the proposition. Condition (ii) of Case I is equivalent to condition b) in the proposition. □

B.2 Proof of Proposition 2

In this case

$$A = \frac{1}{a^*\mu\beta} \begin{bmatrix} \mu\beta + \bar{\sigma}^*\mu\kappa_y^* & -\bar{\sigma}^*\mu \\ -a^*\kappa_y^* & a^* \end{bmatrix},$$

where a^*, $\bar{\sigma}^*$ and κ_y^* are defined in the proposition.

1. Note that under the restriction that $a^* > 0$, $det(A) > 0$. Therefore it cannot be the case that $det(A) - tr(A) < -1$ and $det(A) + tr(A) < -1$. This means that determinacy can only obtain under the conditions of Case I. In addition, if $\varphi < 1$, then $tr(A) > 0$, so $det(A) + tr(A) > -1$. Condition c is therefore obsolete if $\varphi < 1$.

2. For $a^* < 0$, $det(A) < 0$, so Case I cannot hold. The conditions given in the proposition are those pertaining to Case II. □

B.3 Corollary 6 to Proposition 2 and Proof

Corollary 6 *Under the conditions of Proposition 2, the following special cases obtain:*

1. *With no endogenous risk premium ($\xi = 0$), the range of parameters for which the equilibrium is determinate is larger if spending is countercyclical ($\varphi < 0$), rather than acyclical. In addition, the range of fundamental parameters implying determinacy of the equilibrium is smaller, the larger φ.*

2. *With an endogenous risk premium $\xi > 0$, instead, the range of parameters for which the equilibrium is determinate is often larger if spending is procyclical, i.e., if spending is cut during a deep recession. More precisely:*

   (a) If the conditions of Part 1 of Proposition 2 hold as well as $\varphi \in (0, 1)$ and $\phi_{T,y} < 1 - \frac{\kappa\nu}{(1-\beta\mu)\xi}$, then the range of fundamental parameters for which the equilibrium is determinate is at least as large as in the absence of an endogenous response in spending, and can be larger. Note that this case is more likely, the less elastic the tax revenue responds to the state of the economy (the smaller $\phi_{T,y}$), and the more responsive the country's risk premium to the deficit (the larger ξ).

   (b) It may occur that the equilibrium is indeterminate if government spending does not respond to output, but becomes determinate with a mild procyclical response that satisfies $\frac{1+\xi\bar{\sigma}\phi_{T,y}}{1+\xi\bar{\sigma}} < \varphi < 1$ (this is is only case under which the conditions of Case 2 of Proposition 2 can hold). Note that this inequality is more likely satisfied the steeper the risk premium and the less elastic the response of taxes.

Proof.

1. If $\xi = 0$, $a^* = \mu > 0$. As a result condition Case 1 of Proposition 2 gives the relevant condition. First note that condition a) will always be satisfied. Condition c) holds for $\varphi < 1$. What remains to be checked therefore is whether condition b) holds for $\varphi < 0$ whenever it holds for $\varphi = 0$, and holds for some fundamental parameters for which it would not hold otherwise. That is true if

$$(1 - \beta\mu)(1 - a^*) - \mu\bar{\sigma}^*\kappa_y^* > (1 - \beta\mu)(1 - a) - \mu\bar{\sigma}\kappa_y,$$

   or equivalently (for $\xi = 0$),

$$\mu\bar{\sigma}^*\kappa_y^* - \mu\bar{\sigma}\kappa_y < 0.$$

Substituting, the condition reads

$$\mu\kappa\left[\frac{\bar{\sigma}\nu}{1-\varphi}+1\right]-\mu\kappa\left[\bar{\sigma}\nu+1\right]<0.$$

This reduces to $\varphi/(1-\varphi)<0$, which is true for $\varphi<0$. So the range of fundamental parameters for which determinacy obtains is bigger with a countercyclical government spending response in this case than in the absence of any response. What remains to be shown is that a stronger response further increases the range of fundamental parameters for which determinacy obtains. To see this, observe that the left-hand side of condition b) in Case 1 of Proposition 2 is independent of φ. The right-hand side is given by

$$\mu\bar{\sigma}^*\kappa_y^*=\mu\kappa[1+\nu\bar{\sigma}/(1-\varphi)].$$

The right-hand side is strictly increasing in φ. As a result, the set of parameters for which the condition will bind will be the larger the more negative φ is.

2. (a) The range of fundamental parameters for which determinacy holds is bigger if $a^*<a$, and if

$$(1-\beta\mu)(a-a^*)>\mu\bar{\sigma}^*\kappa_y^*-\mu\bar{\sigma}\kappa_y.$$

$a^*<a$ boils down to $\frac{\phi_{T,y}-\varphi}{1-\varphi}<\phi_{T,y}$, which is true for $\varphi<1$. The second condition reduces to

$$(1-\beta\mu)\xi(1-\phi_{T,y})\frac{\varphi}{1-\varphi}>\kappa\nu\frac{\varphi}{1-\varphi}.$$

For $\varphi\in(0,1)$ this yields $\phi_{T,y}<1-\frac{\kappa\nu}{(1-\beta\mu)\xi}$, the condition in the corollary.

(b) $a^*<0$ means $\frac{1+\xi\bar{\sigma}\phi_{T,y}}{1+\xi\bar{\sigma}}<\varphi<1$, so this is the only case in which Part 2 of Proposition 2 can be satisfied.

■

B.4 Proof of Proposition 3

The assumed Markov structure means that output, inflation and government spending (in deviation from the steady state) will take on the same values while the lower bound binds,

y_l, π_l and g_l, respectively, and values of zero thereafter. The IS curve thus implies

$$y_l - g_l = \mu(y_l - g_l) - \bar{\sigma}[-\log(1+i^d) + \Gamma + \mu\xi(g_l - \phi_{T,y}y_l) - \mu\pi_l].$$

And the Phillips curve implies

$$\pi_l = \mu\beta\pi_l + \kappa_y y_l - \kappa_g g_l.$$

Solving these equations for y_l and π_l gives for y_l:

$$y_l = \vartheta_r[\log(1+i^d) - \Gamma] \; + \vartheta_g g_l,$$

where ϑ_r and ϑ_g take on the values given in the proposition. In addition, $\vartheta_r > 0$: the numerator is positive, and the denominator is positive, too, by condition b) for determinacy in Proposition 1. □

B.5 Corollary 7 to Proposition 3 and Proof

Corollary 7 *Under the parameter restrictions of Proposition 1:*

1. *The government spending multiplier, ϑ_g, is positive if and only if*

$$(1-\mu) - \frac{\mu\bar{\sigma}\kappa_g}{1-\beta\mu} > \mu\xi\bar{\sigma}. \tag{45}$$

   Note that, conversely, the spending multiplier will be negative if the risk premium sufficiently affects the economy, i.e., if ξ is large enough.

2. *If $\xi = 0$, provided that the conditions for determinacy in Proposition 1 are satisfied, the government spending multiplier is strictly larger than one. This case corresponds to the analysis by Christiano et al. (2011) and Woodford (2011).*

3. *If $\xi > 0$, the government spending multiplier is unambiguously larger than one if $\phi_{T,y} > 1 - \frac{\kappa\nu}{\xi(1-\beta\mu)}$, that is, if the tax revenue rises sufficiently fast with output. In addition, government spending at the lower bound is self-financing if $\vartheta_g > 1/\phi_{T,y}$.*

Proof.

1. Under the restrictions for determinacy of Proposition 1, the denominator of ϑ_g is unambiguously positive. $\vartheta_g > 0$ thus requires $(1-\beta\mu)(1-b) - \mu\bar{\sigma}\kappa_g > 0$, which solves to the expression in equation (45).

2. The conditions for determinacy require that $(1-\beta\mu)(1-a)-\mu\bar{\sigma}\kappa_y>0$, so the denominator of ϑ_g is positive. The same condition can also be used to prove that the numerator of ϑ_g is positive. Extending the above inequality yields:

$$(1-\beta\mu)(1-b)-\mu\bar{\sigma}\kappa_g>-(1-\beta\mu)(b-a)-\mu\bar{\sigma}(\kappa_g-\kappa_y).$$

   Note that $\kappa_g<\kappa_y$. In addition, note that $b=a$ if $\xi=0$. This proves that $(1-\beta\mu)(1-b)-\mu\bar{\sigma}\kappa_g>0$ if $\xi=0$ and under the conditions of Proposition 1.

3. For $\xi>0$, $\vartheta_g>1$ is equivalent, after substituting for κ_g and κ_y, to $\phi_{T,y}>1-\frac{\kappa\phi}{\xi(1-\beta\mu)}$. The deficit is given by $g-\phi_{T,y}y_l$. Spending will thus be self-financing if $1-\phi_{T,y}\vartheta_g<0$.

■

B.6 Proof of Proposition 4

For the austerity phase the IS equation is given by

$$(y_a-g_a)(1-\varrho)=-\bar{\sigma}\left[\phi_\pi\pi_a-\varrho\pi_a\right].$$

The Phillips curve is given by

$$\pi_a=\beta\varrho\pi_a+\kappa_y y_a-\kappa_g g_a.$$

These two equations solve to expressions (43) and (44). While at the lower bound, the IS equation is given by

$$y_l(1-\mu)=(1-\mu)(y_a-g_a)-\bar{\sigma}\left[-\log(R)+\Gamma+\xi[(1-\mu)g_a-\phi_{T,y}(1-\mu)y_a-\phi_{T,y}\mu y_l]-\mu\pi_l-(1-\mu)\pi_a\right].$$

The Phillips curve is given by

$$\pi_l=\beta\mu\pi_l+\beta(1-\mu)\pi_a+\kappa y_l.$$

Solving the latter two equations leads to the equation for output, y_l, equation (42).

B.7 Proof of Corollary 5

1. For the case $\xi=0$, abstracting from constants, we have that $y_l=1/d[(1-\mu)(1-\beta\mu)(y_a-g_a)+\bar{\sigma}(1-\mu)\pi_a]$.

Note that $d = (1-\mu)(1-\beta\mu) - \bar{\sigma}\mu\kappa_y > 0$ since Proposition 4 assumes determinacy of the equilibrium; cp. condition b) in Proposition 1. $y_l > 0$ (meaning that future austerity increases output at the ZLB relative to the case of no action) thus requires

$$(1-\mu)(1-\beta\mu)(y_a - g_a) + \bar{\sigma}(1-\mu)\pi_a > 0. \tag{46}$$

Substitute for y_a and π_a using equations (43) and (44). Further note that the denominator in the expressions for π_a and y_a is positive (we have assumed determinacy in Proposition 4, so $\phi_\pi > 1$, and therefore especially $\phi_\pi - \varrho > 0$). Furthermore, observe that $\kappa_y - \kappa_g > 0$, and that $g_a < 0$. Using these observations, inequality (46) resolves to $\varrho > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.

2. Let y_l^w denote the size of output at the lower bound with a response of the risk premium ($\xi > 0$). Denote with a superscript ^{o} the terms in the absence of a response of the risk premium. For example, let y_l^o denote the size of output in the absence of a response of the risk premium ($\xi = 0$).
   Note, first, that y_a, g_a, π_a are independent of the risk premium.
   Note, second, that $d^w = (1-\beta\mu)(1-\mu-\mu\xi\phi_{T,y}\bar{\sigma}) - \bar{\sigma}\mu\kappa_y < d^o$.
   Note, third, that $d^w > 0$ by the assumption of determinacy. Thus $[\frac{d^o}{d^w} - 1] > 0$.
   Condition $y_l^w > y_l^o$, after substituting for y_a and π_a, and after dividing by $g_a < 0$, is equivalent to

$$\begin{aligned}&\left[\tfrac{d^o}{d^w} - 1\right]\bar{\sigma}(1-\mu)(\kappa_g - \kappa_y)\left[(1-\beta\mu)(\phi_\pi - \varrho) - 1 + \varrho\right]\\ &+ \tfrac{d^o}{d^w}\bar{\sigma}(\phi_\pi - \varrho)(\kappa_g - \kappa_y)\bar{\sigma}\xi\phi_{T,y}(1-\mu)(1-\beta\mu)\\ &+(1-\mu)(1-\beta\mu)\left[(1-\varrho)(1-\beta\varrho) + \bar{\sigma}(\phi_\pi - \varrho)\kappa_y\right]\bar{\sigma}\xi(\phi_{T,y} - 1) < 0.\end{aligned}$$

   The second row is nonpositive. The third row is strictly negative (since $\phi_{T,y} \in [0,1)$). $\kappa_g - \kappa_y < 0$, so the first row will be strictly negative if $(1-\beta\mu)(\phi_\pi - \varrho) - 1 + \varrho > 0$, which is equivalent to $\varrho > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.

3. It suffices to show one such parameterization. In particular, let $\phi_{T,y} = 0$. In that case, the sovereign risk channel does not affect the determinacy condition, nor does it affect the denominator defined as d above. The condition that ensures that a retrenchment

after the ZLB has a positive effect on output while at the ZLB is

$$\frac{1}{d}[(1-\mu)(1-\beta\mu)(y_a-g_a)+\bar{\sigma}(1-\mu)\pi_a-d(1-\mu)(1-\beta\mu)\bar{\sigma}\xi g_a]>0$$

Now, y_a, π_a do not depend on ξ. In addition, under the conditions provided in the proposition, $d(1-\mu)(1-\beta\mu)\bar{\sigma}\xi$ is unambiguously positive if $\xi > 0$ while g_a is negative. As a result, for any persistence of the retrenchment ϱ (and so in particular also for those that violate the inequality provided in Item 1), there exist values of $\xi \geq 0$ that make the above condition hold.

C Nonlinear model equations

This section collects all the equations of the model and the fundamental parameters of the economy.

C.1 Consumer and labor supply

Euler equation savers (for deposits)

$$e_t\lambda_t^s = \beta E_t\left[e_{t+1}\frac{1+i_t^d}{\Pi_{t+1}}\left\{(1-\delta)\pi_b\lambda_{t+1}^b+[\delta+(1-\delta)\pi_s]\lambda_{t+1}^s\right\}\right].$$

Euler equation savers (government bonds)

$$e_t\lambda_t^s = \beta E_t\left[e_{t+1}\frac{(1-\vartheta_{t+1})(1+i_t^g)}{\Pi_{t+1}}\left\{(1-\delta)\pi_b\lambda_{t+1}^b+[\delta+(1-\delta)\pi_s]\lambda_{t+1}^s\right\}\right].$$

Euler equation borrowers

$$e_t\lambda_t^b = \beta E_t\left[e_{t+1}\frac{1+i_t^b}{\Pi_{t+1}}\left\{(1-\delta)\pi_s\lambda_{t+1}^s+[\delta+(1-\delta)\pi_b]\lambda_{t+1}^b\right\}\right].$$

Marginal utility of consumption saver

$$c_t^s = \xi^s(\lambda_t^s)^{-\sigma_s}.$$

Marginal utility of consumption borrower

$$c_t^b = \xi^b(\lambda_t^b)^{-\sigma_b}.$$

FOC labor supply saver

$$h_t^s = \left(\frac{\lambda_t^s}{\psi_s}(1-\tau_t^w)w_t\right)^{1/\nu}.$$

FOC labor supply borrower

$$h_t^b = \left(\frac{\lambda_t^b}{\psi_b}(1-\tau_t^w)w_t\right)^{1/\nu}.$$

Aggregate labor supply (definition)

$$h_t = \pi_b h_t^b + (1-\pi_b)h_t^s$$

Definition Λ_t

$$\Lambda_t := \psi\left[\pi_b\left(\frac{\lambda_t^b}{\psi_b}\right)^{1/\nu} + \pi_s\left(\frac{\lambda_t^s}{\psi_s}\right)^{1/\nu}\right]^{\nu}.$$

Definition average marginal utility of consumption:

$$\lambda_t = \pi_b\lambda_t^b + (1-\pi_b)\lambda_t^s.$$

Aggregate private borrowing:

$$\begin{aligned} b_t &= \delta b_{t-1}(1+\omega_{t-1})(1+i_{t-1}^d)/\Pi_t - \pi_b\omega_t b_t + \pi_b\left[\delta b_{t-1}^g(1+i_{t-1}^g)/\Pi_t - b_t^g\right] \\ &\quad + \pi_b\pi_s[(c_t^b - c_t^s) - (1-\tau_t^w)(w_t h_t^b - w_t h_t^s)]. \end{aligned}$$

C.2 Financial intermediation

Definition spread between lending and deposit rates:

$$1+\omega_t = \frac{1+i_t^b}{1+i_t^d}.$$

FOC loan origination:

$$\omega_t = \chi_t + \Xi_t.$$

Fraction of loans lost or resource costs of origination:

$$\chi_t = \chi_\psi[(1+i_t^g)/(1+i_t^d)]^{\alpha_\psi} - 1 \text{ and } \Xi_t = 0, \text{ or } \Xi_t = \chi_\psi[(1+i_t^g)/(1+i_t^d)]^{\alpha_\psi} - 1 \text{ and } \chi_t = 0.$$

C.3 Firms and NKPC

FOC price setting:

$$\left(\frac{P_t^*}{P_t}\right)^{1+\theta(\phi-1)} = \frac{K_t}{F_t}.$$

Definition of K_t:

$$K_t = \lambda_t e_t \mu^p \phi w_t \left(\frac{y_t}{z_t}\right)^{\phi} + \alpha\beta E_t\left[\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta\phi} K_{t+1}\right].$$

Definition of F_t:

$$F_t = \lambda_t e_t y_t + \alpha\beta E_t\left[\left(\frac{\Pi_{t+1}}{\Pi}\right)^{(\theta-1)} F_{t+1}\right].$$

The law of motion for prices (inflation):

$$1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1} = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{1-\theta}.$$

Price dispersion:

$$\Delta_t = \alpha\Delta_{t-1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta\phi} + (1-\alpha)\left(\frac{1-\alpha\left(\Pi_t/\Pi\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}.$$

C.4 Government

Real government debt:

$$b_t^g = \frac{b_{t-1}^g(1+i_{t-1}^g)}{\Pi_t} + g_t - \frac{T_t^g}{P_t} - \tau_t^w w_t h_t.$$

Part of tax revenue related to the business cycle and stabilization policy:

$$tr_t := \tau_t^w w_t h_t + T_t^g/P_t.$$

Tax rule

$$(tr_t - tr) = \left[\phi_{T,y}(y_t - y) + \phi_{T,b^g}(b_{t-1}^g - b^g)\right],\ \phi_{T,y} \geq 0,\ \phi_{T,b} > 0.$$

In addition, we need to specify a rule for τ_t^w. Indeed, for most of the paper, $\tau_t^w = 0$.

Ex ante probability of a default, p_t, at a given level of indebtedness, b_t^g,

$$p_t = F_{\text{beta}}\left(\frac{b_t^g}{4y}\frac{1}{\overline{b}^{\text{g,max}}}; \alpha_{b^g}, \beta_{b^g}\right).$$

Haircut

$$\vartheta_t = \begin{cases} \vartheta_{\text{def}} & \text{with probability } p_t, \\ 0 & \text{with probability } 1 - p_t. \end{cases}$$

Taylor rule:

$$\log(1 + i_t^{d,*}) = \log(1 + i^d) + \phi_\Pi \log(\Pi_t/\Pi) - \phi_\omega \log((1 + \omega_t)/(1 + \omega)).$$

$i_t^d = \max\{i_t^{d,*}, 0\}$.

C.5 Market clearing

Demand for final goods is given by

$$y_t = \pi_b c_t^b + \pi_s c_t^s + g_t + \Xi_t \, b_t$$

Supply of final goods is given by

$$y_t \Delta_t^{1/\phi} = z_t h_t^{1/\phi}.$$

C.6 Summary: Model variables

Exogenous variables: e_t, g_t, z_t.

Endogenous variables: b_t, b_t^g, c_t^b, c_t^s, χ_t, Δ_t, F_t, h_t, h_t^b, h_t^s, i_t^b, i_t^d, $i_t^{d,*}$, i_t^g, K_t, λ_t, λ_t^b, λ_t^s, Λ_t, ω_t, p_t, P_t^*/P_t, Π_t, τ_t^w, T_t^g/P_t, tr_t, ϑ_t, w_t, Ξ_t, y_t.

C.7 Summary: Fundamental parameters

α: Calvo stickiness.

α_{b^g}: first parameter in the distribution of the "fiscal limit."

α_ψ: spillover parameter sovereign spread to private-sector spread.

β : time-discount factor.

β_{b^g}: second parameter in the distribution of the "fiscal limit."

$\overline{b}^{g,max}$: third parameter in the distribution of the "fiscal limit."

χ_ψ : scaling parameter interest spread borrower-lender.

δ : persistence of type

μ_p : gross price markup, $\mu_p = \theta/(\theta - 1)$.

ν : inverse of Frisch elasticity.

ϕ: elasticity of hours with respect to output.

ϕ_Π: Taylor rule response to inflation.

ϕ_ω: Taylor rule response to private-sector spread.

$\phi_{T,y}$: response of taxes to output.

$\phi_{T,b}$: response of taxes to debt.

π_b: fraction of borrowers

π_s : fraction of savers

ψ_b: scaling constant disutility of work borrower.

ψ_s : scaling constant disutility of work saver.

σ_s: IES saver

σ_b : IES borrower

θ : elasticity of demand.

ϑ_{def} : haircut

ξ^b: scaling parameter utility of consumption borrower.

ξ^s: scaling parameter utility of consumption saver.

D Linearized model

This section collects the linearized model equations. For the sake of brevity of exposition, we drop the expectations operator. It is implicitly understood that all terms carrying a $t+1$ index refer to the expectations as of period t of those variables. Also, in the derivations we impose that output in the steady state equals unity, $y = 1$.

D.1 Consumer and labor supply

Euler equation savers (for deposits)

$$\widehat{e}_t + \widehat{\lambda}_t^s = \widehat{e}_{t+1} + \widehat{i}_t^d - \widehat{\Pi}_{t+1} + (1-\chi_s)\widehat{\lambda}_{t+1}^b + \chi_s \widehat{\lambda}_{t+1}^s.$$

Euler equation savers (government bonds)

$$\widehat{e}_t + \widehat{\lambda}_t^s = \widehat{e}_{t+1} + \widehat{i}_t^g - \widehat{\Pi}_{t+1} - \frac{p\vartheta_{\text{def}}}{1-p\vartheta_{\text{def}}}\,\widehat{p}_{t+1} + (1-\chi_s)\widehat{\lambda}_{t+1}^b + \chi_s \widehat{\lambda}_{t+1}^s.$$

Euler equation borrowers

$$\widehat{e}_t + \widehat{\lambda}_t^b = \widehat{e}_{t+1} + \widehat{i}_t^b - \widehat{\Pi}_{t+1} + \chi_b \widehat{\lambda}_{t+1}^b + (1-\chi_b)\widehat{\lambda}_{t+1}^s.$$

Marginal utility of consumption saver

$$\widehat{c}_t^s = -\sigma_s \widehat{\lambda}_t^s.$$

Marginal utility of consumption borrower

$$\widehat{c}_t^b = -\sigma_b \widehat{\lambda}_t^b.$$

FOC labor supply saver

$$\widehat{h}_t^s = \frac{1}{\nu}\left[\widehat{\lambda}_t^s + \widehat{\tau}_t^w + \widehat{w}_t\right].$$

FOC labor supply borrower

$$\widehat{h}_t^b = \frac{1}{\nu}\left[\widehat{\lambda}_t^b + \widehat{\tau}_t^w + \widehat{w}_t\right].$$

Aggregate labor supply (definition)

$$h\widehat{h}_t = \pi_b \, h^b \, \widehat{h}_t^b + (1-\pi_b)\, h^s \, \widehat{h}_t^s.$$

Implied FOC for aggregate labor supply (redundant)

$$\widehat{h}_t = \frac{1}{\nu}\left[\widehat{\Lambda}_t + \widehat{\tau}_t^w + \widehat{w}_t\right].$$

Definition Λ_t

$$\widehat{\Lambda}_t = \gamma_b \widehat{\lambda}_t^b + (1-\gamma_b)\widehat{\lambda}_t^s.$$

Definition average marginal utility of consumption:

$$\widehat{\lambda}_t = \pi_b \frac{\lambda^b}{\lambda}\widehat{\lambda}_t^b + (1-\pi_b)\frac{\lambda^s}{\lambda}\widehat{\lambda}_t^s$$

Aggregate private borrowing:

$$\begin{aligned}
\tilde{b}_t \quad = \quad & \delta(1+\omega)(1+i^d)/\Pi\, \tilde{b}_{t-1} \\
& +\delta b(1+\omega)(1+i^d)/\Pi \left[\widehat{\omega}_{t-1} + \widehat{i}^d_{t-1} - \widehat{\Pi}_t\right] \\
& -\pi_b \omega \tilde{b}_t - \pi_b b(1+\omega)\widehat{\omega}_t \\
& +\pi_b \delta(1+i^g)/\Pi\, \tilde{b}^g_{t-1} \\
& +\pi_b \delta b^g (1+i^g)/\Pi \left[\widehat{i}^g_{t-1} - \widehat{\Pi}_t\right] - \pi_b \tilde{b}^g_t \\
& +\pi_b \pi_s \left[(c^b \widehat{c}^b_t - c^s \widehat{c}^s_t) - w(h^b - h^s)\widehat{\tau}^w_t - (1-\tau^w)\left(w(h^b-h^s)\widehat{w}_t + wh^b\widehat{h}^b_t - wh^s\widehat{h}^s_t\right)\right],
\end{aligned}$$

Auxiliary equation:

$$\widehat{\Omega}_t := \widehat{\lambda}^b_t - \widehat{\lambda}^s_t :$$

Law of motion for $\widehat{\Omega}_t$:

$$\widehat{\Omega}_t = \widehat{\omega}_t + \bar{\delta}\, \widehat{\Omega}_{t+1}.$$

D.2 Financial intermediation

Definition spread between lending and deposit rates:

$$\widehat{\omega}_t = \widehat{i}^b_t - \widehat{i}^d_t.$$

FOC loan origination:

$$(1+\chi)\widehat{\omega}_t = \widetilde{\chi}_t \text{ or } (1+\Xi)\widehat{\omega}_t = \widetilde{\Xi}_t.$$

(Note: if $\chi > 0$, e.g., and so $\Xi = 0$, we have $\widehat{\omega}_t = \widehat{\chi}_t$, where $\widehat{\chi}_t := \log(1+\chi_t) - \log(1+\chi)$)

Fraction of loans lost/intermediation costs :

$$\widetilde{\chi}_t = (1+\chi)\,\alpha_\psi \left(\widehat{i}^g_t - \widehat{i}^d_t\right) \text{ or } \widetilde{\Xi}_t = \Xi\,\alpha_\psi \left(\widehat{i}^g_t - \widehat{i}^d_t\right).$$

D.3 Firms and NKPC

New Keynesian Phillips curve:

$$\widehat{\Pi}_t = \beta\widehat{\Pi}_{t+1} + \kappa\left[(\phi(1+\nu) + 1/\bar{\sigma} - 1)\widehat{y}_t - \phi(1+\nu)\widehat{z}_t - 1/\bar{\sigma}[\tilde{g}_t + \Xi\,\tilde{b}_t] - \widehat{\tau}^w_t + [s_\Omega + \pi_b - \gamma_b]\widehat{\Omega}_t\right].$$

Price dispersion

$$\widehat{\Delta}_t = 0.$$

D.4 Government

Real government debt:

$$\widetilde{b}_t^g = \frac{(1+i^g)}{\Pi}\tilde{b}_{t-1}^g + \frac{b^g(1+i^g)}{\Pi}\left[\widehat{i}_{t-1}^g - \widehat{\Pi}_t\right] + \tilde{g}_t - \widetilde{tr}_t.$$

Tax rule

$$\widetilde{tr}_t = \phi_{T,y}\widehat{y}_t + \phi_{T,b^g}\,\tilde{b}_{t-1}^g.$$

Ex ante probability of a default, p_t, at a given level of indebtedness, b_t^g,

$$\tilde{p}_t = f_{\text{beta}}\left(\frac{b^g}{4y}\frac{1}{\overline{b}^{\text{g,max}}};\alpha_{b^g},\beta_{b^g}\right)\frac{1}{4y}\frac{1}{\overline{b}^{\text{g,max}}}\,\tilde{b}_t^g,$$

where f_{beta} is the pdf of the beta distribution.

Taylor rule:

$$\widehat{i}_t^{d,*} = \phi_\Pi\widehat{\Pi}_t - \phi_\omega\widehat{\omega}_t.$$

$\widehat{i}_t^d = \max\left\{\widehat{i}_t^{d,*}, -(1+i^{\,d,*})\right\}.$

D.5 Market clearing

Demand for final goods is given by

$$\widehat{y}_t = \pi_b\, s_b\, \widehat{c}_t^b + \pi_s\, s_s\, \widehat{c}_t^s + \tilde{g}_t + \Xi\,\tilde{b}_t + b\,\widetilde{\Xi}_t.$$

Supply of final goods is given by

$$\widehat{y}_t = \widehat{z}_t + \frac{1}{\phi}\widehat{h}_t.$$

D.6 Definition of linearized variables

Exogenous variables:

$\widehat{e}_t = \log(e_t/1).$

$\tilde{g}_t = g_t - g.$

$\widehat{z}_t = \log(z_t/z).$

Endogenous variables:

$\tilde{b}_t = b_t - b.$

$\tilde{b}_t^g = b_t^g - b^g.$

$\widehat{c}_t^b = \log(c_t^b/c^b),$

$\widehat{c}_t^s = \log(c_t^s/c^s),$

$\widetilde{\chi}_t = \chi_t - \chi.$

$\widehat{\Delta}_t = \log(\Delta_t/\Delta),$

$\widehat{h}_t = \log(h_t/h),$

$\widehat{h}_t^b = \log(h_t^b/h^b),$

$\widehat{h}_t^s = \log(h_t^s/h^s),$

$\widehat{i}_t^b = \log(1+i_t^b) - \log(1+i^b)$

$\widehat{i}_t^d = \log(1+i_t^d) - \log(1+i^d)$

$\widehat{i}_t^{d,*} = \log(1+i_t^{d,*}) - \log(1+i^d)$

$\widehat{i}_t^g = \log(1+i_t^g) - \log(1+i^g)$

$\widehat{\lambda}_t = \log(\lambda_t/\lambda),$

$\widehat{\lambda}_t^b = \log(\lambda_t^b/\lambda^b),$

$\widehat{\lambda}_t^s = \log(\lambda_t^s/\lambda^s),$

$\widehat{\Lambda}_t = \log(\Lambda_t/\Lambda),$

$\widehat{\omega}_t = \log(1+\omega_t) - \log(1+\omega).$

$\widehat{\Omega}_t := \widehat{\lambda}_t^b - \widehat{\lambda}_t^s.$

$\widehat{p}_t = \log(p_t) - \log(p)$

$\widehat{\Pi}_t = \log(\Pi_t) - \log(\Pi)$

$\widehat{\tau}_t^w = \log(1-\tau_t^w) - \log(1-\tau^w).$

$\widetilde{tr}_t = tr_t - tr.$ $\widehat{\vartheta}_t = \log\left([p_{t+1}(1-\vartheta_{\text{def}}) + [1-p_{t+1}]/[1-p\,\vartheta_{\text{def}}]\right).$

$\widehat{w}_t = \log(w_t/w),$

$\widetilde{\Xi}_t = \Xi_t - \Xi,$

$\widehat{y}_t = \log(y_t/y).$

E Definition of auxiliary parameters

$\bar{\delta}$:

$$\bar{\delta} = \chi_b + \chi_s - 1.$$

χ_b:

$$\chi_b = \frac{\beta(1+i^b)}{\Pi}[\delta + (1-\delta)\pi_b].$$

χ_s:

$$\chi_s = \frac{\beta(1+i^d)}{\Pi}[\delta + (1-\delta)\pi_s].$$

γ_b :

$$\gamma_b = \pi_b \left(\frac{\lambda_b}{\psi_b} / \frac{\Lambda}{\psi}\right)^{1/\nu} = \frac{\pi_b \left(\frac{\lambda_b}{\psi_b}\right)^{1/\nu}}{\pi_b \left(\frac{\lambda_b}{\psi_b}\right)^{1/\nu} + \pi_s \left(\frac{\lambda_s}{\psi_s}\right)^{1/\nu}} \overset{\text{hours FOCs}}{=} \frac{\pi_b}{\pi_b + \pi_s \frac{h^s}{h^b}}.$$

κ :

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{1}{1+\theta(\phi-1)}.$$

κ_y :

$$\kappa_y = \kappa \left(\phi(\nu+1) + \frac{1}{\overline{\sigma}} - 1\right).$$

κ_g :

$$\kappa_g = \kappa \frac{1}{\overline{\sigma}}.$$

ψ, scaling constant "aggregate disutility of work":

$$\psi^{-1/\nu} = \pi_b \psi_b^{-1/\nu} + \pi_s \psi_s^{-1/\nu}.$$

ψ_Ω :

$$\Psi_\Omega = \pi_b(1-\chi_b) - \pi_s(1-\chi_s).$$

$\overline{\sigma}$:

$$\overline{\sigma} = \pi_b s_b \sigma_b + \pi_s s_s \sigma_s.$$

$s_b = c^b/y$, $s_s = c^s/y$.

s_Ω :

$$s_\Omega = \frac{\pi_b \pi_s [s_b \sigma_b - s_s \sigma_s]}{\overline{\sigma}}.$$

F Three-equation representation of the aggregate economy

The following IS curve can be derived for the linearized model

$$
\begin{aligned}
\widehat{y}_t \quad = \quad & \widehat{y}_{t+1} - (\tilde{g}_{t+1} - \tilde{g}_t) - \left[\Xi(\tilde{b}_{t+1} - \tilde{b}_t) + b\,(\tilde{\Xi}_{t+1} - \tilde{\Xi}_t)\right] \\
& -\overline{\sigma}\left[\widehat{i}^{\,d}_t - \widehat{\Pi}_{t+1} + \widehat{\Gamma}_t + (\pi_b + s_\Omega)\widehat{\omega}_t - [\psi_\Omega + s_\Omega(1-\bar{\delta})]\widehat{\Omega}_{t+1}\right],
\end{aligned}
$$

where

$$
\widehat{\Omega}_t = \widehat{\omega}_t + \bar{\delta}\,\widehat{\Omega}_{t+1}.
$$

With the New Keynesian Phillips curve being as above:

$$
\begin{aligned}
\widehat{\Pi}_t \quad = \quad & \beta\,\widehat{\Pi}_{t+1} \\
& +\kappa\left[(\phi(1+\nu) + 1/\bar{\sigma} - 1)\widehat{y}_t - \phi(1+\nu)\widehat{z}_t - 1/\bar{\sigma}[\tilde{g}_t + \Xi\,\tilde{b}_t + b\,\tilde{\Xi}_t] - \widehat{\tau}^{w}_t + [s_\Omega + \pi_b - \gamma_b]\widehat{\Omega}_t\right].
\end{aligned}
$$

Taylor rule:

$$
\widehat{i}^{\,d,*}_t = \phi_\Pi \widehat{\Pi}_t - \phi_\omega \widehat{\omega}_t.
$$

$\widehat{i}^{\,d}_t = \max\left\{\widehat{i}^{\,d,*}_t, -(1 + i^{\,d,*})\right\}$. The three-equation system, together with an assumption about the evolution of the spread $\widehat{\omega}_t$, describes the evolution of output, inflation and interest rates (independently of private debt) if

1. $\Xi = 0$, that is, no aggregate resource costs of loan origination.

2. $[\psi_\Omega + s_\Omega(1-\bar{\delta})] = 0$.

3. $\widehat{\tau}^{w}_t = 0$, that is, distortionary taxes don't move or are nonexistent.

4. $[s_\Omega + \pi_b - \gamma_b] = 0$.

The parameters and laws of motion chosen in the core of the paper satisfy these conditions.

G Linking ξ and ξ'

This appendix provides the foundation for equation (38) in the main text. The formula is motivated through a sequence of back-of-the-envelope calculations. Focus on the interest rate spread in the IS curve, neglecting other terms. Assume that initially, in period t, the economy is at the lower bound. This gives a relationship of

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \bar{\sigma}\widehat{\varpi}_t + ...$$

Iterating forward, we have

$$\tilde{y}_t = -\bar{\sigma} E_t \left\{ \sum_{j=0}^{\infty} \widehat{\varpi}_{t+j}\, I(\text{economy at ZLB in period } t+j) \right\},$$

where $I()$ is the indicator function. If the spread depends on the future debt level, as in the full version of the model presented in Section 2, we have

$$\widehat{\varpi}_{t+j} = \xi'\, E_{t+j} \tilde{b}^g_{t+j+1}$$

so

$$\tilde{y}_t = -\bar{\sigma} E_t \left\{ \sum_{j=0}^{\infty} I(\text{economy at ZLB in period } t+j)\; \xi'\; E_{t+j} \tilde{b}^g_{t+j+1} \right\}.$$

Abstracting from effects of interest payments and the valuation of debt, $\tilde{b}^g_{t+j+1}$ is roughly the sum of an initial debt level in $t-1$ (assumed to be zero without loss of generality in the following exposition) and the deficits accumulated in period t through $t+j+1$. Let $deficit$ be the deficit. Following the Markov structure, we assume that the structural primary deficit (before extraordinary debt-stabilization measures) is the same in every period at the ZLB. With the same abstraction as above, conditional on being at the ZLB in $t+j$,

$$E_{t+j}\, \tilde{b}^g_{t+j+1} = \tilde{b}^g_{t+j} + \mu \cdot deficit = (j+1+\mu)\, deficit.$$

Then,

$$\tilde{y}_t = -\bar{\sigma}\xi'\, E_t \left\{ \sum_{j=0}^{\infty} I(\text{economy at ZLB in period } t+j)(j+1+\mu) deficit \right\}.$$

And so

$$\tilde{y}_t = -\bar{\sigma}\, \xi'\, deficit \sum_{j=0}^{\infty} \mu^j (j+1+\mu),$$

or, equivalently.

$$\tilde{y}_t = -\bar{\sigma}\, \xi' \frac{1+\mu(1-\mu)}{(1-\mu)^2}\; deficit. \tag{47}$$

In contrast, the analytical version of the model in Section 3 has

$$\tilde{y}_t = -\bar{\sigma} E_t \left\{ \sum_{j=0}^{\infty} I(\text{economy at ZLB in period } t+j)\xi \; deficit \; E_{t+j}[\, I(\text{at ZLB in } t+j+1)] \right\},$$

which boils down to

$$\tilde{y}_t = -\bar{\sigma}\, \xi \sum_{j=0}^{\infty} \mu^j \; deficit \; \mu.$$

Or, equivalently,

$$\tilde{y}_t = -\bar{\sigma}\, \xi \, \frac{\mu}{1-\mu} \; deficit. \tag{48}$$

Comparing (47) and (48) leads to the relationship in the main text, namely, equation (38):

$$\xi = \frac{1+\mu(1-\mu)}{\mu(1-\mu)} \, \xi'.$$