DOI: 10.1111/jmcb.12521

DENIS PELLETIER
CENGIZ TUNC

Endogenous Life-Cycle Housing Investment and Portfolio Allocation

This paper develops a life-cycle portfolio allocation model to address the effects of housing investment on the portfolio allocation of households. The model employs a comprehensive housing investment structure, Epstein–Zin recursive preferences, and a stock market entry cost. Furthermore, rather than resorting to calibration we estimate the value of the relative risk aversion and elasticity of intertemporal substitution. The model shows that housing investment has a strong crowding out effect on investment in risky assets throughout the life-cycle. We further find that the effect of the presence of housing investment on households portfolio allocation is larger than the effect of having EZ recursive preferences.

JEL codes: D10, G0, G11, G12
Keywords: portfolio choice, housing, life-cycle model, real estate.

Low stock market participation rates and moderate equity holdings for stock market participants are two important empirical observations in the U.S. data. For instance, the 2007 Survey of Consumer Finance (SCF) shows that only 55.3% of U.S. households have direct or indirect holdings of risky assets.[1] Furthermore, data from the Panel Study of Income Dynamics (PSID) for the 1968–2007

We would like to thank the editor Pok-sang Lam and anonymous referees for their constructive suggestions. We also thank Douglas Pearce, John Seater, seminar participants at NC State macroeconomics workshop, Central Bank of the Republic of Turkey, ECB Fourth Conference on Household Finance and Consumption, Bogazici University Center for Economics and Econometrics 2012 Annual Conference, Midwest Finance Association 2016 Annual Conference, and Istanbul Bilgi University Conference on Advances in Applied Macro-Finance and Forecasting for helpful comments. This paper is part of the doctoral dissertation of Cengiz Tunc at NC State University.

Denis Pelletier *is an Associate Professor in the Department of Economics, North Carolina State University* (*E-mail*: denis_pelletier@ncsu.edu). Cengiz Tunc *is an Assistant Professor at the Department of Economics, Toros University* (*E-mail*: cengiz.tunc@toros.edu.tr).

Received February 8, 2016; and accepted in revised form May 2, 2018.

1. Risky assets include tax-deferred accounts, directly held stocks, directly held pooled investment funds, bonds, and managed investment accounts or equity in a trust or annuity. Tax-deferred retirement accounts consist of both personally established individual retirement accounts (IRAs) and job-based 401(k) accounts. For detailed information, see Bucks et al. (2009).

Journal of Money, Credit and Banking, Vol. 00, No. 0 (xxxx 2018)

period show that the median household direct risky asset holdings and indirect risky asset holdings are zero. Despite this fact, theoretical models with the assumption of the historically prevailing equity premium predict that almost 100% of households should hold risky assets as part of their financial portfolio. This gap between theoretical predictions and empirical observations still poses a great challenge to life-cycle models.

While the primary investment asset for U.S. households is investment in owner-occupied housing, it is generally ignored in portfolio allocation models. It is quite logical that a typical household has a higher priority to invest in housing in order to have an owner-occupied house than investing in the stock market.[2] This fact is probably the main reason for the low stock market participation observed in the data. Cocco (2005) shows that due to the investment in housing, younger and poorer households have less wealth to invest in the stock market. Furthermore, Vestman (2013) shows substantial difference in portfolio allocation of homeowners and renters.

The focus of our research is twofolds. First, this paper incorporates housing investment into a life-cycle asset allocation model to provide an explanation for these two empirical observations: low stock market participation rates and moderate equity holdings for stock market participants. Specifically, we investigate the effects of housing investment on portfolio allocation of households in a life-cycle model through the construction of a fairly comprehensive structural model. This involves solving and simulating the life-cycle asset allocation model and estimating some crucial parameters. Second, we analyze the effects of having Epstein–Zin (EZ) recursive preferences on households' portfolio allocation.[3] The EZ recursive preferences provide the flexibility to disentangle the risk aversion from intertemporal substitution. The main drawback of the commonly used additive utility functions such as the constant relative risk aversion (CRRA) utility function is that relative risk aversion (RRA), which gives information about how agents deal with uncertainty across possible states of the world, is the inverse of the elasticity of intertemporal substitution (EIS), which is just time preference. In other words, CRRA utility imposes two different roles on the same parameter whereas EZ-type preferences disentangle this relationship.

Housing is different from other financial assets because it serves dual benefits. First, it is a durable consumption good from which owners obtain utility. Second, it also serves as an investment asset that enables owners to hold home equity. Contrary to liquid financial assets such as bonds and stocks, housing investment is illiquid and often highly leveraged. While housing plays an important role in portfolio allocation, it is largely unexplored in the literature because of the difficulty of dealing with various frictions associated with the housing market, such as homeowner/renter distinction, mortgage payment, liquidation cost, moving decision, etc.

2. According to the latest release (2014Q3) of the Census Bureau, the homeownership rate in the United States is 64.3%. The average homeownership rate between 1980Q1 and 2014Q3 is 65.5% with standard deviation of 1.72.

3. See Epstein and Zin (1989, 1991) for details of these recursive preferences.

Beside housing, our model incorporates some key features in order to better explain the asset allocation profiles of households. The first feature is the use of EZ preferences where the RRA is disentangled from the EIS. The second feature is that stock market participation is endogenously done in each period with an entry cost to the market for the first-time investment (i.e., buying risky assets). The entry cost is widely accepted in the literature even though too little investigation has been done on its magnitude.[4] A computationally easy way of introducing an entry cost is considering it as a fixed proportion of annual labor income as in Gomes and Michaelides (2008), Guvenen (2009b), Guo (2004), and Alan (2006). This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, and acquiring and evaluating information about the stock market plus the opportunity cost of time. The third feature is that labor supply is inelastic and households receive uninsured labor income in each working period. We use PSID data from 1980 to 2007 to realistically calibrate the life-cycle labor income process of households. Finally, we incorporate a bequest motive in our model which assumes that households bequest all their financial investments and investments made in housing to their inheritors when they pass away.

This comprehensive model is able to explain the aforementioned empirical observations stated above. More importantly, the model is able to explain them by successfully estimating suitable RRA and EIS parameters. We obtain both moderate stock market participation rate as well as moderate equity holding among the stock market participants. Hence, housing investment incorporated into a life-cycle asset allocation model has strong crowding out effects on investment in the risky asset when compared to the no-housing model. The effects are significant throughout the life-cycle.

We further compare the portfolio allocations under the EZ recursive preferences with estimated risk aversion and EIS parameters to the portfolio allocations under the standard CRRA utility function with estimated parameter of RRA.[5] The results show that under the EZ recursive preferences the life-cycle portfolio allocation profiles matches the empirical evidence better than the same profiles under the CRRA case.

The other results are as follows. The model generates reasonable estimates for the homeownership rate during the working period and a slight overestimate for the retirement period. The model also shows that renters have higher share of their financial wealth in the risky asset than homeowners mainly because while homeowners can enjoy the return on their housing investment and hence do not need to rely heavily on risky financial investments, renters need to invest more in the risky financial investment as it is the investment through which they can accumulate wealth.

Households wealth allocation throughout the life-cycle has been analyzed in the portfolio allocation literature since the pioneering studies of Merton (1969) and Samuelson (1969). Although it has been ignored by a huge body of the portfolio

4. For example, see Alan (2006) and Khorunzhina (2013) for a detailed analysis of the entry cost.

5. Note that under CRRA utility assumption EIS is the inverse of the RRA. Hence, the estimated risk aversion under CRRA automatically gives an estimate for EIS.

allocation literature over a very long period of time, there is a gradually growing literature that treats housing as an important determinant of portfolio allocation. Leung (2004) provides a comprehensive literature review of housing and asset pricing. Grossman and Laroque (1990) develop an asset allocation model where infinitely lived households derive utility from a single indivisible durable consumption good. They argue that an adjustment cost for illiquid assets could answer the equity premium puzzle. Flavin and Nakagawa (2008) extend the Grossman and Laroque (1990) model by including both durable and nondurable consumption goods into the utility function. Using a continuous-time framework, the paper compares a housing model to a habit persistence model and finds that while both deliver many of the same implications, empirical tests using household level data strongly favor the housing model. Longstaff (2009) studies the implications of illiquid assets in a continuous time asset pricing exchange economy with heterogeneous agents. Fernández-Villaverde and Krueger (2011) present a general equilibrium model of life-cycle asset allocation to demonstrate the effects of consumer durable goods on consumption and asset allocations. Piazzesi, Schneider, and Tuzel (2007) consider a consumption-based asset pricing model where housing is explicitly incorporated into the model both as an asset and as a consumption good. Their paper focuses on the effects of housing-consumption asset pricing models on the predictability of the return on stocks.

The closest articles to our research are Hu (2005), Cocco (2005), and Yao and Zhang (2005). Hu (2005) develops a standard life-cycle portfolio allocation model that provides the flexibility for households to endogenously decide whether to be a renter or a homeowner. The model employs a CRRA utility function with two sets of preference parameters (renters and homeowners) and considers only five time periods where each period corresponds to either 10 or 15 years. Although the paper obtains low levels of investment in the stock market relative to standard models with no housing, the investment in the stock market is still significantly higher than the empirically observed values. Cocco (2005) analyzes portfolio choice in the presence of housing in a standard life-cycle model with a CRRA utility function. Each time period in this model corresponds to 5 years. The model in that paper assumes that all agents are homeowners. There is no endogenous decision to be a renter or homeowner. The paper concludes that (i) house price risk crowds out stockholding, (ii) households have a relatively low stock market participation rate compared to standard models with no housing, and (iii) younger and poorer investors have limited financial wealth to invest in the stock market. However, it is less successful at matching the share of wealth invested in stocks conditional on participation with predicted values much higher than those observed in the data. The last paper similar to ours is Yao and Zhang (2005). The model in this paper is similar to Cocco (2005) except it allows households to decide between renting or owning a house. Results of Yao and Zhang (2005) show that renters invest a higher portion of their financial wealth in the stock market than homeowners. Furthermore, the share of wealth invested in the stock market is greatly higher than empirical observations.

Our research differs from these studies in several dimensions. First, we develop a comprehensive life-cycle portfolio allocation model that incorporates all features introduced by different studies in the portfolio allocation literature. Among these are housing as an investment and a durable consumption good, an endogenous decision on being a renter or homeowner, a stock market entry cost and an endogenous decision on stock market investment, EZ-type preferences, and a bequest motive. Second, the life-cycle portfolio allocation papers that incorporate housing into the model generally calibrate the parameters of the model. Instead, we estimate two crucial parameters: the RRA and EIS. To our knowledge, this paper is the first one in the life-cycle housing-portfolio allocation literature to estimate parameters instead of resorting to calibration. Finally, we explicitly show that the life-cycle conditional risky asset shares better match the data when we use EZ recursive preferences (i.e., disentangling the RRA from the EIS) than using the CRRA assumption (i.e., impose the restriction that RRA is the inverse of the EIS).

The rest of the paper is organized as follows. Section 1 presents the model with all features and defines the optimization problem. The parameterization of the model constitutes Section 2. We discuss estimation of the RRA and EIS parameters in Section 3. Section 4 presents the results of the model, comparative static analysis and comparison of the results of the model with the data. Section 5 provides concluding remarks. The exposition of the solution technique of the life-cycle model is relegated to an Appendix.

1. MODEL

1.1 Household Preferences

This model is a discrete time life-cycle model where each period corresponds to 1 year. As a general convention in the life-cycle literature each year is actually the real age of a household minus 19. We assume that households live for at most T periods. The probability that a household is alive at age t conditional on being alive at age $t-1$ is q_t.

In each period, a household derives utility from a constant elasticity of substitution (CES) utility function with nondurable consumption goods, C, and housing investment (or consumption), H. Preferences are in the form of EZ, where the RRA is disentangled from the EIS:

$$V_t = \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left(\mathrm{E}_t \left[q_{t+1} V_{t+1}^{1-\gamma} + (1-q_{t+1}) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \tag{1}$$

$$\theta = \frac{1-\gamma}{1-1/\psi}, \tag{2}$$

$$u\left(C_t, H_t\right) = \left[C_t^{v} + H_t^{v} \right]^{\frac{1}{v}}, \tag{3}$$

where β is the time discount factor, γ is the RRA parameter, and ψ is the EIS parameter. The intratemporal elasticity of substitution between nondurable consumption goods and housing investment is $1/(1-\upsilon)$. W_{t+1} is the total wealth that a household would bequest if it passes away at age t.

1.2 Labor Income Process

During their working life, households supply labor inelastically in each period and receive stochastic labor income Y_{it}. Define $y_{it} = \log(Y_{it})$, where y_{it} has the following process:

$$y_{it} = f(t, Z_{it}) + u^l_{it}, \tag{4}$$

$$f(t, Z_{it}) = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 gender + \beta_4 marital\_status + \beta_5 educ, \tag{5}$$

where Y_{it} is labor income received by household i at age t, $f(t, Z_{it})$ is a deterministic function of age t and household characteristics Z_{it} (education, marital status, and gender). Following Cocco (2005), we assume that the shock to the log of labor income u^l_{it} is composed of both aggregate (η^l_t) and idiosyncratic (ε^l_{it}) components. We assume that the idiosyncratic component is transitory and i.i.d normally distributed with mean 0 and variance $\sigma^2_{\varepsilon^l}$ and the aggregate shock has an AR(1) process:

$$\eta^l_t = \rho \eta^l_{t-1} + w^l_t, \tag{6}$$

where w^l_t is also i.i.d normally distributed with mean 0 and variance $\sigma^2_{w^l}$. Therefore, during the working period, log labor income is the sum of a hump-shaped deterministic component and two random components, one transitory and one persistent. For simplicity, we assume that the retirement age is deterministic. Households work until period K where K corresponds to an age of 65 ($K = 46$). During the retirement periods ($t > K$), households receive a constant and deterministic labor income $Y_{it} = \xi Y_{iK}$ where $0 < \xi < 1$ (a fraction of their income during the last year before retiring).[6]

1.3 Housing Investment

Households enter the market as renters in the first period. From the second period on, they endogenously decide either to buy a house and become homeowners or stay in their current house as renters. A typical homeowner at any time can endogenously decide either (i) to stay in his current house, or (ii) to sell his current house and buy a new one, or (iii) to sell his house and become a renter. Similarly, a renter can decide either (i) to stay in current rented house, or (ii) to rent a new house, or (iii) to buy a house and become a homeowner. Housing investment, by its nature, is a

6. During the whole retirement period, u_{it} becomes u_{iK}.

lump-sum and illiquid investment. Buying a house requires paying a proportion d of the house market value as downpayment and financing the rest through a mortgage. To capture the illiquidity of housing investment, households incur a liquidation cost equal to a fraction κ of the house market value. Homeowners pay an amount equal to a proportion δ of the house's market value for maintenance and depreciation expenses in each period. Renters, on the other hand, do not pay for maintenance and depreciation costs; they only pay an annual rent which is equal to a proportion α_R of the house's market value. House-related expenses are calculated as a proportion of the house's market value in the literature because it is easy in this way to incorporate them into the model and reduces the computational burden.

Per unit price of housing is denoted by P_t^h, such that a house of size H_i has price $P_t^h H_i$ at time t. We define p_t^h as the log of house price, $p_t^h = \ln(P_t^h)$, and assume that it follows the following stochastic process:

$$\Delta p_t^h = \mu_h + \varepsilon_t^h, \tag{7}$$

where Δp_t^h follows a stochastic process with the average growth rate of house prices is μ_h.

When a household buys a house of size H_i at time t, it pays the downpayment which is equal to $dP_t^h H_i$ and finances the rest through a mortgage with a fixed rate r^m. RM_t denotes the amount of mortgage debt a household has at age t. When buying a house, households can borrow up to the house value minus the downpayment:

$$RM_t \leq (1-d)P_t^h H_i. \tag{8}$$

Housing investment constitutes an important part of households wealth. However, because of its illiquidity, this investment cannot be directly used for consumption purpose. As households pay back their mortgage debt over time, their housing investment (home equity) and total wealth increases while liquid wealth does not necessarily increase. Once households decide to sell their houses by paying the transaction cost, then they transform the housing investment into liquid wealth that can be used for consumption.

1.4 Financial Assets and Wealth Accumulation

There are only two financial assets that households can invest in: a risky asset and a riskless asset. Return on investment in the risky asset follows the following stochastic process as in Campbell et al. (2001), Cocco (2005), and others:

$$R_{t+1}^s - R^b = \mu_s + \varepsilon_{t+1}^s, \tag{9}$$

where R_{t+1}^s is the gross return on the risky asset at time $t+1$, R^b is the constant gross return on the riskless asset, μ_s is the excess return from the risky asset over the riskless one (equity premium), and ε_{t+1}^s is distributed as $N(0, \sigma_{\varepsilon^l}^2)$. In order to invest in the risky asset, households are required to pay a one-time fixed entry cost. This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, the cost of acquiring and evaluating information about the risky asset, and the opportunity cost of time. Alan (2006) estimates this cost as approximately 2% of annual labor income while Haliassos and Michaelides (2003) obtain a wide range of this cost from 3% to 24% of annual labor income and Gomes and Michaelides (2008) calibrates this fixed cost at 6% of annualized labor income. Unlike the risky asset, there is no cost for investing in the riskless asset.

Wealth accumulation of renters. In this model, there are two types of households: renters and homeowners. Depending on their homeownership status, households have different budget constraints and cash-on-hand structures. The liquid wealth (LW) of a typical household who was renter at age $t-1$ is the sum of investments made at age $t-1$ in both risky (S) and riskless (B) assets:

$$LW_t = R^b B_{t-1} + R_t^s S_{t-1}. \tag{10}$$

We refer to the sum of liquid wealth and labor income as cash-on-hand. At age t, the cash-on-hand is $X_t \equiv LW_t + Y_t$. A typical renter uses this cash-on-hand to decide whether to buy a house by paying the downpayment and begin to pay annual mortgage payments or just stay in a rental property and pay an annual rent. He also decides on consumption expenditure, whether to pay the fixed entry cost for investment in the risky asset (if it has not been paid yet) and decides the portfolio composition among the different financial assets:

$$X_t = C_t + S_t + B_t + FIX_t \alpha_F Y_t + (1 - HR_t)\left[\alpha_R P_t^h H_t\right] + HR_t\left[M_t^n + dP_t^h H_t\right], \tag{11}$$

where S_t is the total investment made in the risky asset and B_t is the total investment made in the riskless asset at age t. FIX_t is a dummy variable that takes the value of 1 if a renter pays the fixed entry cost at age t, and 0 otherwise. The term $\alpha_F Y_t$ represents the fixed entry cost which is the proportion α_F of the annualized labor income at age t. HR_t is a dummy variable that takes the value of 1 if a household become homeowner at time t and it is 0 otherwise. The term M_t^n is the household's annual mortgage payment at age t for a newly purchased house. The structure of the mortgage payments is explained in details in the next subsection, which describes the budget constraint of homeowners.

Wealth accumulation of homeowners. The liquid wealth structure of a typical homeowner at age t is the same as the liquid wealth structure of a renter. Similarly, the cash-on-hand of homeowners at age t is the sum of liquid wealth and labor income $X_t \equiv LW_t + Y_t$. A typical homeowner decides in every period whether to stay in his current house or sell the current house and buy another one or sell the current house and move to a rental property. Once he sells his house, he would use his cash-on-hand plus the proceeds from selling the house to either buy a new house (upgrade or downgrade) by paying the downpayment or move to a rental property and pay rent.

He would then decide on consumption expenditure and investment in financial assets using the remaining cash-on-hand. On the other hand, if he decides to stay in the current house, then he would pay the annual mortgage payment (if the mortgage debt is not yet paid back) and use the remaining cash-on-hand for consumption expenditure and investments in financial assets. Furthermore, households pay for the maintenance and depreciation expense which is equal to proportion δ of the house's market value. It follows that the budget constraint of households takes the following form:

$$\begin{aligned} X_t = {} & C_t + S_t + B_t + FIX_t\alpha_F Y_t + (1 - MS_t)\left[M_t^o + \delta P_t^h H_{t-1}\right] \\ & + HR_t MS_t\left[dP_t^h H_t + M_t^n + \delta P_t^h H_{t-1} - \left((1-\kappa)P_t^h H_{t-1} - RM_t\right)\right] \\ & + (1 - HR_t)MS_t\left[\alpha_R P_t^h H_t - \left((1-\kappa)P_t^h H_{t-1} - RM_t\right)\right], \end{aligned} \tag{12}$$

where MS_t is a dummy variable that takes the value of 1 if a homeowner moves to a new house or rental property and 0 if he stays in his current house. The term $(1-\kappa)P_t^h H_{t-1}$ denotes the amount a homeowner receives if he sells his house, adjusted for the liquidation cost. RM_t is the total mortgage debt a homeowner has at age t and M_t^o is the annual mortgage payment at age t for a previously purchased house, which consists of both the annual interest payment MI_t and the annual principal payment MP_t. The mortgage debt and annual mortgage payments have the following processes. In each period, MI_t is calculated from remaining mortgage balance:

$$M_t^o = MP_t + MI_t, \tag{13}$$

$$RM_t = RM_{t-1} - MP_t. \tag{14}$$

The mortgage payments, including the first payment M_t^n on a newly purchased house, are set such that the mortgage is paid back over a period of 25 years, given the mortgage rate r^m.

Households are assumed to satisfy the nonnegativity constraints on consumption and housing investment and short-sale constraints on the risky and the riskless asset. They cannot have negative amounts of consumption and housing investment. Furthermore, households cannot borrow at the riskless rate in order to invest in the risky asset.

$$C_t \geq 0, \qquad S_t \geq 0, \qquad B_t \geq 0, \qquad H_t \geq 0, \qquad \forall t. \tag{15}$$

Furthermore, we define the correlations between (i) the shocks to returns on investment in the risky asset and the persistent shocks to labor income as ρ_{sl}, (ii) the shocks to returns on investment in the risky asset and the shocks to returns on housing prices as ρ_{sh}, and finally (iii) the shocks to returns on housing investment and the persistent shocks to labor income as ρ_{hl}.

It is common in the recent life-cycle portfolio allocation models to incorporate a bequest motive in order to match the skewness of the wealth distribution (e.g., see, Gourinchas and Parker 2002, Laitner 2002, Gomes and Michaelides 2005, Yao and Zhang 2005). Conditional on being alive at age $t-1$, a household could pass away with probability $1-q_t$. We assume that households bequest their total liquid wealth and all investments made in housing to their inheritors at any period they pass away:

$$W_t = R^b B_{t-1} + R_t^s S_{t-1} + P_t^h H_{t-1} - RM_t. \tag{16}$$

1.5 Optimization Problem

Before defining the optimization problem and the value function, we list the state and the control variables in a compact form. The state variables are age (t), liquid wealth (LW_t), risky asset participation status ($IFIX_t = 1$ if the fixed cost has been paid, $= 0$ otherwise), homeownership status ($O_t = 1$ if homeowner, $= 0$ otherwise), the size of house owned from the previous period (H_{t-1}), the remaining mortgage debt (RM_t), and the lagged labor income shock. We denote the state variables by Ω, where $\Omega_t = \{t, LW_t, IFIX_t, O_t, H_{t-1}, RM_t, \eta_{i,t-1}\}$. The control variables are consumption (C_t), moving decision (MS_t), homeownership decision (HR_t), the size of house a household chooses at age t (H_t), the risky asset participation decision (FIX_t), and the share of financial investment in the risky asset (s_t). We denote the control variables by Ψ, where $\Psi_t = \{C_t, MS_t, HR_t, H_t, FIX_t, s_t\}$. Given the stochastic labor income, stochastic house price evolution and stochastic risky asset returns, the household's optimization problem is then

$$V_t(\Omega_t) = \max_{\Psi_t} \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left(\mathrm{E}_t \left[q_t V_{t+1}(\Omega_{t+1})^{1-\gamma} + (1-q_t) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{17}$$

subject to dynamics, restrictions and budget constraints in equations (4)–(16).

2. PARAMETERIZATION

Two crucial parameters (RRA and EIS) will be estimated but we must calibrate the remaining parameters of the model. Table 1 displays the list of parameters that are calibrated and their values.

Each period in the model corresponds to 1 year. Households are assumed to enter the market at age 20 and live for at most 80 additional years. The age-dependent labor income process is based on PSID data from 1980 to 2007. In order to obtain a random sample of U.S. households, we dropped families that were part of the Survey of Economic Opportunities.[7] We treat retirement period labor income as a constant fraction of labor income of the last working period as in Gomes and Michaelides (2005)

7. The original PSID sample was drawn from two independent sources: an oversample of roughly 2,000 poor families selected from the Survey of Economic Opportunities (SEO), and a nationally representative

TABLE 1

BASELINE PARAMETER VALUES

Description	Parameter	Value
Time discount factor	β	0.95
Gross return on the riskless asset	R^b	1.03
Equity premium	μ_s	0.06
Liquidation cost	κ	0.10
Intratemporal elasticity of substitution	ν	0.33
Rental rate	α_R	0.06
Mortgage rate	r^m	0.03
Fixed entry cost	α_F	0.05
Downpayment	d	0.20
Depreciation and maintenance	δ	0.01
Average growth rate on housing prices	μ_h	0.01
Std. of persistent shock to labor income	$\sigma^2_{w^l}$	0.1632
Std. of temporary shock to labor income	$\sigma^2_{\varepsilon^l}$	0.3272
Std. of shocks to return on housing inv.	σ_h	0.057
Std. of shocks to return on risky asset inv.	σ_s	0.20
Retirement income factor	ξ	0.66
Persistence parameter of labor income shocks	ϕ	0.82
Correlations between shocks		
- return on housing and return on risky asset inv.	ρ_{sh}	0.20
- return on housing investment and labor income	ρ_{hl}	0.075
- return on risky asset inv. and labor income	ρ_{sl}	0.10

and Cocco (2005). The permanent component of labor income, equation (5), is a function of households' age and age squared, as well as dummy variables for education (1 if the head of the household has at least 12 years of education, 0 if not), marital status (1 if married and 0 if not), and gender (1 if head of household is male and 0 if female). We define labor income widely enough to account for endogenous ways of self-insurance against labor income shocks. Labor income includes total labor income, unemployment compensation, social security, total transfers, child support, and other welfare of both the head of the family and his spouse if present. Following the procedure in Cocco (2005), we estimate the standard deviations of both aggregate and transitory shocks and the persistence parameter of labor income using the same PSID data from 1980 to 2007. We estimate the persistence parameter ϕ as 0.82 which is in line with the findings of Guvenen (2009a) and estimate the standard deviation of w^l_t and ε^l_{it} at 0.1632% and 0.3272%, respectively. These estimates are in line with the parameters in Campbell et al. (2001) and Cocco (2005), and Cocco, Gomes, and Maenhout (2005).

We calibrate several parameters related to housing. Yao and Zhang (2005) and Hu (2005) set the required downpayment rate at 20% of the house value while Cocco (2005) sets it at 15%. We set the downpayment rate as 20% of the house market value. To be consistent with previous studies, we set the annual rental rate of houses at 6% of the market value of the rental property, and annual maintenance and depreciation

sample of roughly 3,000 families from all states. Since the SEO is oversample of low-income families, it is not a random sample of the U.S. population.

cost at 1% of the house's market value.[8] We set $v = 0.33$ so that the intratemporal elasticity of substitution between nondurable consumption and housing, $1/(1-v)$, is 1.5 as in Piazzesi, Schneider, and Tuzel (2007).

We next discuss the value of the parameters related to housing prices in equation (7). In order to obtain annual growth rate of house prices, we estimate equation (7) using the Case–Shiller index. The annual average real growth rate of housing prices is 1.9% for the Case–Shiller index with a standard deviation of 5.72%. However, part of the price increase is due to quality improvement which cannot be accounted for using Case–Shiller index. Therefore, we use 1% instead of 1.9% in order to take into account the increase due to quality changes of houses. Accordingly, we set μ_h =0.01 and $\sigma_h = 0.0572$.

We set the time discount factor β to 0.95. The real risk-free rate R^b is 1.03. We set the annual mean return on the risky asset at 9% which means a 6% equity premium μ_s with a standard deviation of 20%.

There is no consensus in the literature on the size of correlations between shocks to the returns on the risky asset, persistent shocks to labor income, and housing price shocks. Flavin and Yamashita (2002) use the PSID data and find that there is almost no correlation between return on investment in the risky asset and the rate of growth of housing prices. Although the authors find that the city-level Case–Shiller index conflicts with the PSID results, they claim that the PSID data are nationwide while city level data are not and so set $\rho_{sh} = 0$. Cocco (2005), Hu (2005), and Yao and Zhang (2005) also assume ρ_{sh} is 0. However, in the baseline model we assume at annual frequency a 0.20 positive correlation between shocks to return on risky asset and shocks to housing investment return. The correlation between persistent shocks to labor income and shocks to the return on investment in the risky asset ρ_{sl} is calibrated in Campbell et al. (2001) to be positive while ranging from 0.3 to 0.5 depending on education of households. However, it is set to zero in Cocco (2005), Hu (2005), and Yao and Zhang (2005). We set the correlation coefficient between persistent shocks to labor income and shocks to the return on investment in the risky asset to 0.10. Finally, we assume 0.075 correlation between persistent shocks to labor income and shocks to the return on housing investment ρ_{hl} in the baseline case. The same correlation is calibrated to 0.5 in Cocco (2005) and 0.2 in Yao and Zhang (2005), while Hu (2005) assumes no correlation.

In general, the mortgage rate is higher than the constant return on the riskless asset because mortgages bear a long-term interest rate risk and a default risk. Since our model does not have interest rate risk and default risk, we set the interest rate on mortgage loans equal to the return on the riskless asset at 3%. This implies that paying down the mortgage debt by an amount x is equivalent to increasing riskless asset investment by an amount x. Housing investment (home equity) increases by less than amount x because part of the mortgage payments goes to the

8. According to Housing Statistics of the United States, in 2000 the median price specified housing units in the United States is $119,600 and monthly median rent is $602. Then a rough calculation shows an annual rent of 6.1% of house value.

interest component of the payment. Households pay their mortgage debt over 25 years and the mortgage payment variables (MP_t, MI_t) are computed accordingly. The one-time fixed-entry cost for investment in the risky asset is set at 5% of annual labor income. Finally, we parameterized the conditional survival probabilities q_t from the mortality tables produced by the National Center for Health Statistics (http://www.cdc.gov/nchs/data/nvsr/nvsr58_21.pdf, Table 1).

There is no analytical solution to this problem. Therefore, we use a numerical approximation based on the value function iteration, starting from the last period T and moving backward in time. The details are given in the Appendix.

3. ESTIMATION

Solving this structural model numerically is extensively time-consuming, so we are able to estimate only two parameters: the RRA γ and the EIS ψ. The remaining parameters are calibrated to the values discussed above. These two parameters play an important role in obtaining life-cycle asset allocation profiles; different values of these two parameters lead to different life-cycle allocations. Furthermore, there is no clear consensus in the literature on the exact values of these parameters. For instance, Vissing-Jorgensen (2002) suggests that limited asset market participation is important for the estimation of the EIS. She estimates the EIS for stockholders to be around 0.3–0.4 and for bond holders around 0.8–1. On the other hand, Guvenen (2006) uses an EIS for stockholders close to 1 while for nonstockholders close to 0.1. Hall (1988) finds that the EIS is unlikely to be much higher than 0.1. However, Hansen and Singleton (1982) and Vissing-Jorgensen and Attanasio (2003) estimate the EIS as greater than 1. They also estimate RRA between 5 and 10. In a life-cycle portfolio allocation model, Gomes and Michaelides (2005) uses different values of RRA and EIS and conclude that life-cycle portfolio allocations are very sensitive to the values of RRA and EIS. Because these two parameters play critical roles and because there is no consensus in the literature on the exact values for them, it is important to estimate these parameters.

We estimate our model using a minimum distance estimator. See, for example, Greene (2008, Chapter 15). Let us denote by $w_{i,t}$ the value taken by variable i at age t over the life-cycle. Let us also denote by $g_{i,t}(\gamma, \psi)$ the predicted value generated by the model for variable i at age t for a given value of the parameters γ and ψ.[9] A consistent estimator of the RRA and EIS parameters is then given by

$$(\hat{\gamma}, \hat{\psi}) = \arg\min_{\gamma,\psi} \sum_{i=1}^{N} \sum_{t=1}^{T} (w_{i,t} - g_{i,t}(\gamma, \psi))^2, \tag{18}$$

where N is the number of variables used in the minimum distance estimation.

9. We simulate a very large number of life-cycle paths (50,000) and use the average as the predicted value.

This estimation procedure is similar to the impulse-response matching approach where the estimates are chosen by minimizing the distance between the empirical impulse-response function and the impulse-response function generated by a model (see Inoue and Kilian [2013] and Hall et al. [2012] and the numerous references therein). Although similar, it is more simple because we directly match the data ($w_{i,t}$) and the prediction from the model. We do not have to deal with the complications associated to the estimation of the empirical impulse-response function.

The life-cycle data we use for the estimation are investment in the risky asset as a share of total financial assets, risky asset market participation rate, and homeownership rate because these three are the main life-cycle profiles we are interested in. We use SCF 2007 for the data because SCF is probably the most comprehensive survey on U.S. households financial assets.

We define total investment in risky assets as the sum of stock mutual funds, bond mutual funds, mortgage-backed bonds, corporate bonds, publicly traded stocks, and foreign bonds. On the other hand, riskless asset are composed of checking accounts, certificate of deposits, government-backed bond mutual funds, U.S, Government bonds, saving bonds, municipal bonds, and cash and call money accounts. For some assets, households are asked how they are invested: either in risky, or riskless, or split between them. Depending on the answers we classify IRA-Keogh accounts, pensions and saving-money market accounts either risky, riskless, or split between both.

Data used for the estimation include households that report all demographic information (i.e., age, gender, marital status, and education background) and necessary data for the estimation. We average-out the data for each age and further smooth-out the lifetime profiles through kernel regression because the raw data are not smooth enough for the estimation. Kernel regression, as a nonparametric regression method, is appropriate when the relation between the dependent and the independent variables is clearly not linear. Define $\bar{y}_t^a$ as the age-dependent averaged data for any one of the variables, then the goal of doing a kernel regression is to estimate the following functional form:

$$\bar{y}_t^a = m(x_t) + e_t, \tag{19}$$

where $m(x_t)$ is not specified and $x_t = t$. To estimate $m(x_t)$, we use a kernel $K_h(x)$ constructed from the normal distribution,

$$K_h(x) = \frac{1}{h\sqrt{2\pi}} e^{-\frac{x^2}{2h^2}}, \tag{20}$$

which yield the following kernel regression estimator:

$$\widehat{m}_h(x_o) = \frac{\sum_{t=1}^{T} K_h(x_t - x_0)\bar{y}_t^a}{\sum_{t=1}^{T} K_h(x_t - x_0)}. \tag{21}$$

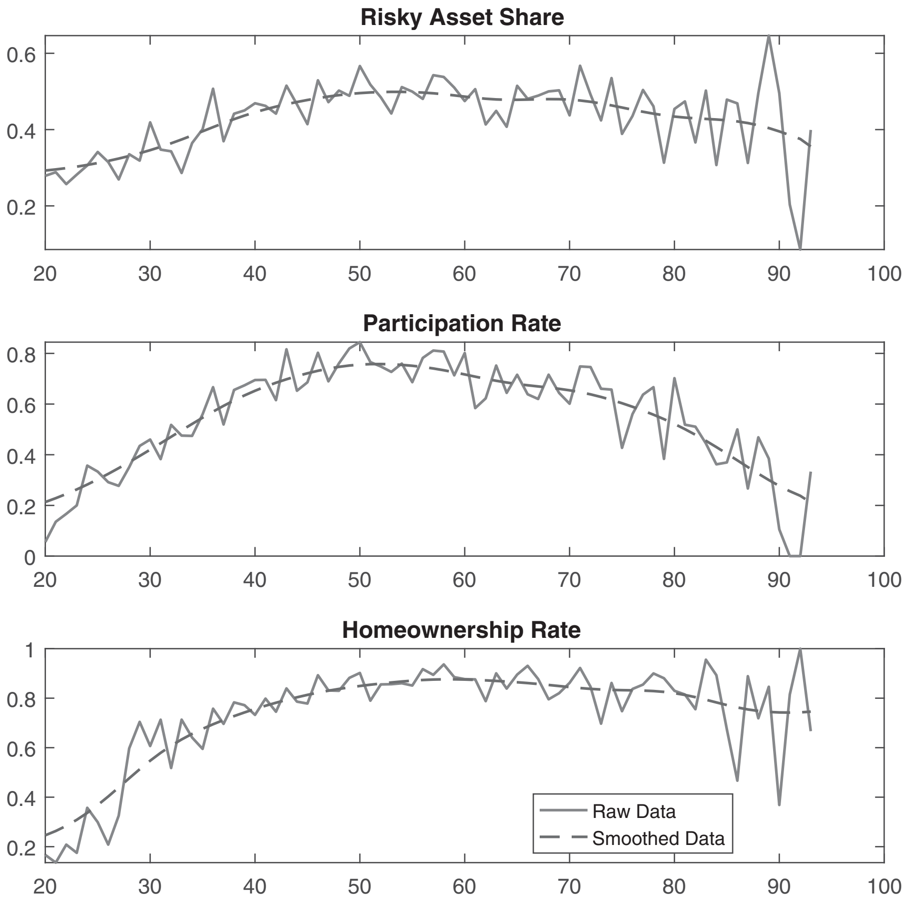

FIG. 1. Raw and Smoothed SCF Data for the Estimation.

We set the bandwidth equal to $h = 0.2\hat{\sigma}_x$ where $\hat{\sigma}_x$ is the sample standard deviation of x_t. We perform this kernel regression separately for each series. The resulting predicted values are used as the observed data $w_{i,t}$ when performing the minimum distance estimation. Figure 1 displays the raw data and the kernel-smoothed data for the estimation.

4. RESULTS

4.1 Model with Housing Investment

The EZ recursive preferences provide us with the flexibility of estimating RRA different than the inverse of the EIS. The estimated value for RRA ($\hat{\gamma}$) and EIS ($\hat{\psi}$) are,

TABLE 2

LIFE-CYCLE PROFILES—BASELINE MODEL AND SCF DATA

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Panel A: Baseline model								
Risky asset share (uncond.)	0.15	0.24	0.41	0.49	0.47	0.37	0.31	0.3
Risky asset share (cond.)	0.33	0.47	0.63	0.66	0.7	0.72	0.65	0.62
Participation rate	0.39	0.51	0.64	0.75	0.68	0.52	0.48	0.49
Homeownership rate	0.32	0.7	0.73	0.74	0.74	0.78	0.86	0.94
Panel B: SCF data								
Risky asset share (uncond.)	0.32	0.38	0.46	0.49	0.46	0.43	0.4	0.38
Risky asset share (cond.)	0.43	0.47	0.52	0.54	0.52	0.52	0.49	0.46
Participation rate	0.52	0.64	0.75	0.8	0.77	0.73	0.64	0.58
Homeownership rate	0.27	0.61	0.78	0.85	0.87	0.84	0.76	0.69

NOTE: Conditional risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset (the risky asset share for the risky asset market participants). In the unconditional risky asset share, we set the risky asset share of households with no risky asset investment to 0. For the SCF data, we define total investment in risky assets as the sum of stock mutual funds, bond mutual funds, mortgage-backed bonds, corporate bonds, publicly traded stocks and foreign bonds and define riskless asset as the sum of checking accounts, certificate of deposits, government-backed bond mutual funds, U.S. Government bonds, saving bonds, municipal bonds, and cash and call money accounts. For some assets, households are asked how they are invested: either in risky, or riskless, or split between them. Depending on the answers we classify IRA-Keogh accounts, pensions and saving-money market accounts either risky, riskless, or split between both. For the last age group, the data come from ages from 91 to 94.

respectively, 3.78 (0.1) and 0.34 (0.24) with the standard errors between parentheses. These estimates are moderate and are in line with the literature. We can also test the hypothesis $H_0 : \gamma = 1/\psi$ against $H_1 : \gamma \neq 1/\psi$ to see if our data are informative enough to be able to support disentangling RRA from the EIS. We can do so by computing a t-statistic, $t = (\hat{\gamma}\hat{\psi} - 1)/se(\hat{\gamma}\hat{\psi})$, where the standard error $se(\hat{\gamma}\hat{\psi})$ is computed with the delta method. We find that $se(\hat{\gamma}\hat{\psi}) = 0.0961$ and $t = 2.9681$. Using a 5% significance level, we can reject $H_0 : \gamma = 1/\psi$ and the assumption of CRRA utility.

Having estimated these parameters within acceptable intervals, we use the corresponding optimal solution to simulate the corresponding life-cycle risky asset and housing investment profiles of households. In Table 2, we report profiles for risky asset share (unconditional and conditional on investing in the risky asset), participation rate in the risky asset market and homeownership rate. Panel A reports average values from simulations of our model. Panel B reports values for these four variables from the SCF data. We use data from the 2007 SCF since SCF provides the most comprehensive information on financial wealth of households in the United States. The data collection process and the definition of each variable is explained in detail in Section 2. Remember that the estimation of γ and ψ is done by matching the life-profile of three of these variables (unconditional risky asset share, participation rate, and homeownership rate). Comparing the results in Panels A and B of Table 2, we see that our model with baseline parameters (Table 1) can match the data reasonably well. We review the results in more details below.

Looking at the results for risky asset share, the first two rows of Panel A show our model predicts that households gradually increase the share of risky asset investment within total financial assets during the working period reaching the highest unconditional share of 49% and highest conditional share of 70% before retirement, before decreasing these shares after retirement. The main reason for the change in the portfolio from high but stochastic return risky asset investment to low and constant return riskless asset investment during the retirement is because as people get older it would be difficult to compensate any big negative shock to risky asset investment. Therefore, they gradually transfer their financial wealth from the risky asset to the riskless one.

One can argue that during early years of working period, it would be optimal to invest the remaining wealth in the risky asset after households make housing investment and consumption decisions. However, there are some reasons that keep households away from investing all remaining wealth only in the risky asset. During the early ages, labor income is the only source of wealth for households and it is low compared to middle-aged households labor income. Once households pay either regular mortgage payment or rent out of labor income, they are left with lower labor income for consumption and financial investment. After the consumption decision, they need to invest some of this labor income in the risk-free asset for precautionary reasons. If they would invest the bulk of the remaining labor income in the risky asset and then face a big negative shock to both labor income and risky asset investment, then they might not be able to pay mortgage payments or rent, or they would be left with too little income for consumption. Therefore, in the early years it is optimal for households to invest the bulk of their financial wealth in the riskless asset and gradually increase the share of risky asset over the working period.

As for the empirical evidence, we see in the first two rows of Panel B that households increase their risky asset investment during the working period and decrease it during the retirement period, a pattern that our model is matching reasonably well. In many previous life-cycle portfolio allocation models, the portfolio shares invested in the risky asset were always significantly higher than the empirical evidence. For example, see Campbell et al. (2001), Cocco (2005), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2008), Hu (2005), Yao and Zhang (2005) among many. The gap between the predictions of life-cycle models and the empirical evidence is called the “portfolio allocation puzzle” which is a kind of flip side of the “equity premium puzzle” of Mehra and Prescott (1985). Therefore, it is important to note that the model is able to match the data fairly well.

The third row of Panel A in Table 2 shows the proportion of the population that has paid the fixed entry cost and is actively investing in the risky asset. The participation rate follows a pattern similar to the pattern of the risky asset share. During the working life, this share monotonically increases from 39% to 75% and then decreases during the retirement period to less than 50%. In the empirical counterpart that can be found in the third row of Panel B, we also see an increasing participation rate during the working period while a decreasing rate for the retirement period. The model prediction is fairly close but lower than the empirical counterpart throughout the life-cycle.

The baseline model finally shows that households have a strong preference for homeownership. In the last row of Panel A, Table 2, we see our model predicts a continuously increasing homeownership rate from 32% to 94% over the life-cycle. Compared to the corresponding SCF data in Panel B, we see that this is a fairly accurate result for the working period but our model overestimates homeownership rate for the late years of the life-cycle.[10] We can think of at least two reasons why our model would overestimate homeownership for older people. First, the model does not include compulsory moving that can happen, for example, because of health reasons, job relocation, or divorce. The compulsory moving is high for retired people while it is low for the working-age people. Second, the model does not include reverse mortgaging which could lead some households to cash in on their house at some point in time during retirement.

Overall, in Table 2 comparing the model-implied results in Panel A with the corresponding SCF data in Panel B, it is clear that the model is able to generate a moderate level of risky asset market participation rate and moderate share of risky asset holding for the whole life-cycle. As we will show later, this is mainly due to housing investment and slightly due to the assumption of EZ recursive preferences, through which we are able to disentangle the RRA from the EIS.

While our primary variables of interest are the ones in Table 2, we also compare the wealth accumulation profiles implied by our model and the corresponding SCF data. Panel A of Table 3 displays the wealth accumulation profiles for the baseline model and for the empirical data.[11] The model shows that households' financial wealth increases during the working life and decreases during the retirement period. It is simply because once households retire, their labor income decreases substantially and they use the investment in financial accounts to compensate the decrease in the labor income and to have a smooth consumption profile over the life-cycle. Total wealth also shows a similar picture during the working period but stays roughly constant during the retirement period as households do not cash in on their home equity or are not allowed to use reverse mortgage in the model. The lower part of Panel A shows the wealth accumulation profiles in the SCF data over the life-cycle. Similarly to the model, we can see that in the SCF data households accumulate both financial wealth and total wealth during the working period and decumulate to some extent during the retirement period.

Panel B of Table 3 displays the differences between the wealth accumulation profiles of risky asset market participants and nonparticipants, implied by the model (B1, B3) and in the SCF data (B2, B4). Both in the model and the data the total wealth and the financial wealth of participants are higher than that of nonparticipants. However, in the model the nonparticipants have more investment in housing (more home equity) than the participants because in the absence of risky asset, nonparticipants prefer to have more investment in the housing investment to compensate, to some

10. For instance, in Yao and Zhang (2005) the homeownership rate is estimated to be 100% from age of 35 to 95.

11. All variables in the SCF data are normalized by mean labor income.

TABLE 3
WEALTH ACCUMULATION PROFILES

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Panel A: Full sample								
A1. Baseline model								
Total wealth	0.85	2.23	3.81	5.19	6.01	6.12	6.31	6.38
Financial wealth	0.22	0.41	0.95	1.63	2.28	2.21	2.07	1.87
A2. SCF data								
Total wealth	0.11	0.65	1.97	3.29	4.18	4.83	4.52	2.96
Financial wealth	0.03	0.36	1.34	2.36	3.08	3.69	3.58	2.45
Panel B: Risky asset market participation status								
B1. Participants (model)								
Total wealth	0.98	2.33	3.63	5.05	6.07	6.5	7.04	7.25
Financial wealth	0.46	0.75	1.44	2.12	3.37	4.21	4.27	3.83
B2. Participants (SCF data)								
Total wealth	0.13	0.75	2.17	3.57	4.73	5.63	5.35	3.62
Financial wealth	0.04	0.42	1.48	2.56	3.51	4.35	4.29	3.02
B3. Nonparticipants (model)								
Total wealth	0.76	2.12	4.15	5.59	5.95	5.72	5.64	5.56
Financial wealth	0.03	0.06	0.09	0.17	0.17	0.06	0.06	0.03
B4. Nonparticipants (SCF data)								
Total wealth	0.03	0.08	0.15	0.60	0.44	0.36	0.62	0.32
Financial wealth	0.00	0.00	0.04	0.33	0.21	0.06	0.24	0.16
Panel C: Homeownership Status								
C1. Homeowners (model)								
Total wealth	1.77	2.82	4.33	5.68	6.33	6.31	6.31	6.32
Financial wealth	0.21	0.28	0.45	0.88	1.29	1.32	1.39	1.53
C2. Homeowners (SCF data)								
Total wealth	0.29	0.98	2.32	3.55	4.32	4.84	4.56	3.02
Financial wealth	0.06	0.54	1.58	2.54	3.16	3.59	3.58	2.45
C3. Renters (model)								
Total wealth	0.22	0.78	2.28	3.74	5.05	5.4	6.27	7.3
Financial wealth	0.22	0.78	2.28	3.74	5.05	5.4	6.27	7.3
C4. Renters (SCF data)								
Total wealth	0.02	0.14	0.89	1.82	3.32	4.33	2.56	1.63
Financial wealth	0.02	0.14	0.89	1.82	3.32	4.33	2.56	1.63

NOTE: Panel A compares the wealth accumulation profiles of the baseline model and the SCF data whereas, Panels B and C display wealth accumulation profiles in the model and the data depending on the risky asset participation and homeownership status. Total Wealth is the sum of financial wealth and home equity, where the financial wealth is composed of investment in risky and riskless financial assets. For the SCF data, we define total financial asset as the sum of stock mutual funds, bond mutual funds, mortgage-backed bonds, corporate bonds, publicly traded stocks and foreign bonds, checking accounts, certificate of deposits, government-backed bond mutual funds, U.S. Government bonds, saving bonds, municipal bonds, cash and call money accounts, IRA-Keogh accounts, pensions and saving-money market accounts. Home equity is the difference between home value and the remaining mortgage debt. In the SCF data, for the last age group, the data come from ages from 91 to 94. These reported wealth numbers are normalized by mean labor income.

extent, the lack of risky asset investment. The financial wealth of the nonparticipants is quite low as they can only invest in the riskless asset with relatively low return. A feature of the SCF data that the model cannot capture is the extremely low level of nonfinancial wealth for nonparticipants.

Finally, in Panel C of Table 3, we compare the wealth accumulation profiles of homeowners and renters. Homeowners are wealthier than renters almost throughout the whole life-cycle both in the model and the data. In the model, due to their housing investment, homeowners do not need to invest as heavily in the risky asset as renters.

TABLE 4

Life-Cycle Profiles—No Housing Investment

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Risky asset share (uncond.)	0.75	0.96	0.98	0.98	0.96	0.83	0.67	0.53
Risky asset share (cond.)	0.77	0.97	0.98	0.98	0.96	0.88	0.78	0.68
Participation rate	0.88	1	1	1	1	0.94	0.86	0.78

Note: Conditional risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset (the risky asset share for the risky asset market participants). In the unconditional risky asset share, we set the risky asset share of households with no risky asset investment to 0.

However, in the SCF data financial wealth of homeowners and renters are close to each other.

4.2 Model without Housing Investment

Next, we compare the aforementioned model to the same model with no housing investment (NHI) in order to quantitatively analyze and measure the effects of housing investment on portfolio allocation. First, it is worth making clear the main features of the NHI model. The NHI model includes all features of the main model except those related to housing investment. In the NHI model, there is no housing investment, no house price risk, no mortgage payments, and no homeownership status. All other parameter values are the same, including RRA and EIS. Households make decisions on consumption expenditure and on investment in financial markets (riskless and risky assets). Table 4 shows the life-cycle portfolio allocation profiles of households for the NHI model.

There are significant changes in life-cycle wealth allocation profiles between these two models. It is clear that housing investment has a strong crowding out effect on the investment in the risky asset. The first row of Table 4 shows that the risky asset market participants invest almost all of their financial wealth in the risky asset during the working period. As the highest share was at most 72% in the baseline case, we observe that housing investment in this setup leads to significant decrease in both conditional and unconditional risky asset investment during the working period. Households reduce their risky asset investment during the retirement in the NHI model similar to the baseline model but the share are still higher than the baseline model.

Compared to the baseline model, we observe higher participation rate during the working period. Between ages of 30 and 70 all households have investment in the risky asset, but the rate decreases during the retirement period. The decrease in the participation rate during the retirement period is simply because households do not want to face any big negative shock to their risky asset investment as they would have less time to compensate big losses.

One reason for a long-lasting crowding out effect of housing investment in the baseline model is the duration of mortgage debt. Most of the households become homeowners and begin to pay their mortgage debt between the ages of 20 and 30. They pay back their mortgage debt over 25 years. When reaching ages around 50, a majority of households are homeowners with relatively little mortgage debt, so they can make investments in financial assets without much concern about their housing investment. However, we also should take into account the fact that some households may want to sell their relatively small house and buy a larger ones. Hence, the negative effects of mortgage debts on financial asset investment could last for an extended period. Furthermore, while the burden of mortgage debt decreases substantially as households get older, they are getting closer to the end of their life and are left with less time to compensate a potential big negative shock to their investment.

4.3 Comparative Statics

In this section, we discuss the effects of some of the parameters of our model on the life-cycle portfolio allocations. Specifically, we investigate the effects of house price risk and the size of the entry cost. We also compare the life-cycle portfolio allocation profiles of homeowners and renters in order to analyze how different these profiles are.

The baseline model assumes that house prices are affected by stochastic shocks. In order to analyze the effect of house price uncertainty on households' portfolio allocation decisions, we set different values for the σ_h ranging from 0% to 20% (in the baseline case we set this value at 5.72%) while keeping the mean real growth rate of house prices constant at 1% as in the baseline case. Table 5 displays the life-cycle portfolio allocation and the homeownership rates for different degrees of house price uncertainty.

Table 5 shows that while keeping the expected return on housing investment constant, as the risk associated with house prices decreases, we observe clear increases in the homeownership rate starting from middle-age forward. For young households, however, we observe the reverse, as housing investment become more risky, relatively many young households are induced to buy a house. The distinction between the behavior of young and nonyoung households on housing investment stems from the fact that young households are more inclined toward the risky assets compared to the other households, while young households perceive more risky housing more appealing asset, nonyoung households perceive less risky housing more appealing asset.

As house prices become more risky, during the first 10 years, the participation rate decreases to some extent as more young households substitute from risky asset investment to housing investment. However, for the subsequent periods, we observe increases in the participation rate as well as the risky asset share within total financial assets due to the fact that housing is now more risky and less appealing asset, and hence households prefer to hold more risky financial assets with higher expected return than risky housing investment with relatively lower expected return.

The baseline case assumes that households pay 5% of the labor income as a one-time fixed entry cost the first time they decide to invest in the risky asset. It covers the

TABLE 5

LIFE-CYCLE PROFILES—HOUSE PRICE RISK

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Panel A: Risky asset share (uncond.)								
Std. of house price								
0.20	0.19	0.48	0.69	0.83	0.82	0.72	0.58	0.46
0.16	0.18	0.35	0.58	0.74	0.73	0.65	0.52	0.42
0.12	0.27	0.27	0.48	0.6	0.58	0.47	0.37	0.31
0.08	0.14	0.23	0.39	0.45	0.43	0.35	0.29	0.29
0.04	0.23	0.29	0.39	0.43	0.4	0.32	0.27	0.26
0.00	0.15	0.25	0.35	0.43	0.37	0.26	0.21	0.21
Panel B: Risky asset share (cond.)								
Std. of house price								
0.20	0.44	0.7	0.84	0.91	0.92	0.91	0.82	0.72
0.16	0.43	0.56	0.76	0.85	0.87	0.88	0.79	0.7
0.12	0.59	0.51	0.7	0.77	0.79	0.81	0.73	0.63
0.08	0.33	0.44	0.64	0.69	0.78	0.88	0.8	0.67
0.04	0.48	0.56	0.66	0.71	0.79	0.92	0.86	0.65
0.00	0.32	0.5	0.63	0.56	0.7	0.91	0.85	0.61
Panel C: Participation rate								
Std. of house price								
0.20	0.36	0.68	0.82	0.91	0.9	0.8	0.71	0.63
0.16	0.38	0.61	0.76	0.87	0.84	0.74	0.66	0.61
0.12	0.42	0.53	0.69	0.78	0.74	0.58	0.51	0.5
0.08	0.39	0.51	0.6	0.65	0.57	0.39	0.36	0.44
0.04	0.43	0.52	0.58	0.6	0.52	0.35	0.31	0.42
0.00	0.41	0.49	0.56	0.77	0.6	0.28	0.25	0.38
Panel D: Homeownership rate								
Std. of house price								
0.20	0.49	0.69	0.46	0.3	0.37	0.52	0.69	0.88
0.16	0.47	0.72	0.56	0.4	0.44	0.56	0.72	0.89
0.12	0.36	0.73	0.64	0.59	0.64	0.77	0.85	0.93
0.08	0.32	0.7	0.73	0.73	0.74	0.79	0.87	0.95
0.04	0.26	0.66	0.73	0.74	0.76	0.82	0.89	0.96
0.00	0.25	0.64	0.77	0.80	0.80	0.85	0.90	0.96

NOTE: Conditional risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset (the risky asset share for the risky asset market participants). In the unconditional risky asset share, we set the risky asset share of households with no risky asset investment to 0.

cost of opening a brokerage account, the cost of understanding how the market works, and the cost of acquiring and evaluating information about the stock market. Similar to Alan (2006), Gomes and Michaelides (2008), Guvenen (2009b), and Guo (2004), this cost is a fixed proportion of households annual labor income. By using different sizes for this cost as a share of labor income, 0%–10%, we analyze its impact on households financial investment decisions. In Table 6, we observe that for this range of entry cost, the results are not sensitive to its size.

We next investigate the life-cycle profiles of homeowners and renters. The model assumes that all households are renters in the initial period. From the second period

TABLE 6

PARTICIPATION RATE FOR DIFFERENT SIZES OF THE ENTRY COST

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Fixed cost								
0	0.44	0.52	0.57	0.61	0.51	0.33	0.3	0.42
0.02	0.43	0.52	0.58	0.61	0.52	0.33	0.3	0.42
0.04	0.43	0.52	0.58	0.61	0.52	0.33	0.3	0.42
0.06	0.43	0.53	0.58	0.62	0.52	0.34	0.3	0.43
0.08	0.43	0.53	0.61	0.64	0.56	0.39	0.35	0.45
0.10	0.42	0.53	0.61	0.65	0.56	0.39	0.35	0.45

NOTE: Entry cost payment rate is the rate of the population that have paid the entry cost.

TABLE 7

BASELINE MODEL LIFE-CYCLE PROFILES—HOMEOWNERS VS. RENTERS

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Homeowners								
Risky share (uncond.)	0.12	0.15	0.21	0.31	0.28	0.21	0.22	0.28
Risky share (cond.)	0.37	0.33	0.4	0.47	0.5	0.54	0.56	0.61
Participation rate	0.31	0.45	0.52	0.65	0.57	0.38	0.39	0.46
Renters								
Risky asset share (uncond.)	0.18	0.47	0.91	1	0.99	0.97	0.85	0.85
Risky asset share (cond.)	0.37	0.7	0.97	1	0.99	0.98	0.85	0.85
Participation rate	0.49	0.66	0.94	1	1	1	1	1

NOTE: Conditional risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset (the risky asset share for the risky asset market participants). In the unconditional risky asset share, we set the risky asset share of households with no risky asset investment to 0.

on and in each period, households endogenously make a homeownership decision. For instance, a homeowner at age t has three options regarding homeownership: (i) stay in the current house, (ii) move to a new house, (iii) move to a rental property. Similarly, a renter decides to either stay in the current rental property or move to a bigger or smaller rental property or buy a house and become a homeowner. Note that a typical household can be a renter for some periods and a homeowner for some other periods. The homeowners part of the table shows the life-cycle profiles for households when they are homeowners. On the other hand, the renter part of the table shows the life-cycle profiles for households when they are renters. For example, assume that a household was a renter for the first 15 years, and endogenously became a homeowner for the next 30 years, and then again endogenously became a renter for the rest of his life. Then, the life-cycle profile for the first 15 years will be used for obtaining renters' life-cycle profiles, the next 30 years will be used to for obtaining homeowners' life-cycle profile, and finally remaining life-cycle profiles will again be used for obtaining renters' life-cycle profile.

Table 7 presents the life-cycle profiles for both homeowners and renters. The results show that renters have a higher participation rate in the risky asset than homeowners. The renters' participation rate increases from 49% to 100% and stays there for the rest of the life-cycle whereas the highest participation rate of homeowners is 65%

around ages between 50 and 60. The unconditional and conditional risky asset shares of renters are also quite higher than that of homeowners throughout the life-cycle. These results are similar to the results in Yao and Zhang (2005) and Hu (2005) who also find that renters' risky asset investment share is larger than homeowners' risky asset investment share. The comparison between homeowners and renters also indicates the crowding out effect of housing investment on households risky asset investment.

As we mentioned earlier, renters are not life-time renters in this model. While some households are renter for some periods, they can endogenously become homeowners for some other periods by paying the downpayment. Therefore, renters portfolio allocation may still be oriented toward home purchases. In order to analyze the impact of the potential of home purchases on renters portfolio allocation, we can compare the portfolio allocation profiles presented in Table 7 to the portfolio allocation households in the no housing case presented in Table 4. In the absence of housing investment, in which households do not expect homeownership in the future, households with age below 60 hold proportionally higher share of risky asset compared to the model in which households do expect potential homeownership in the future.

The gap between these two models for younger adult households is due to the fact that renters with the potential of being homeowner might be saving the downpayment for future home purchase. Therefore, the possibility of being homeowner in the future makes young and middle-aged renters relatively more risk averse in their financial asset investment compared to the people who cannot be homeowner at all.

4.4 Results for Additive Utility

We next investigate the impact of using EZ's recursive utility instead of the more traditional additive utility functions. In the EZ preferences, if we set $\gamma = 1/\psi$ (the RRA being equal to the inverse of the EIS), then it reduces to a CRRA utility function.

Below, we report lifetime profiles for the CRRA utility function. We re-estimate the parameters of the model with CRRA restriction. The estimated parameter in this case is $\gamma = 3.14$ ($\psi = 1/\gamma = 0.3185$). The aim of this case is to see the contribution of disentangling the risk aversion from the inverse of intertemporal substitution. In other words, we want to see the contribution of having EZ-type recursive preferences on households life-cycle portfolio allocation.

Comparing Table 8 to the baseline case in Panel A of Table 2, the unconditional and conditional risky asset shares and the participation rate under the CRRA assumption are higher than the shares under the EZ recursive preferences in the baseline case. Furthermore, comparing to the data in Panel B of Table 2 the unconditional and conditional risky shares under the EZ preferences in the baseline model match the empirical evidence better than the same shares under CRRA utility assumption. On the other hand, the CRRA model has estimates for the homeownership rate that are similar to the EZ case.

TABLE 8

LIFE-CYCLE PROFILES—CRRA WITH $\gamma = 3.14$ AND $\psi = 0.3185$

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Risky asset share (uncond.)	0.16	0.36	0.54	0.65	0.59	0.44	0.47	0.51
Risky asset share (cond.)	0.35	0.61	0.72	0.75	0.75	0.75	0.8	0.86
Participation rate	0.4	0.58	0.74	0.87	0.78	0.59	0.58	0.6
Homeownership rate	0.28	0.6	0.75	0.72	0.7	0.76	0.77	0.93

NOTE: Conditional risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset (the risky asset share for the risky asset market participants). In the unconditional risky asset share, we set the risky asset share of households with no risky asset investment to 0.

5. CONCLUSION

In this study, we focus on the joint effects of housing investment and EZ recursive preferences on portfolio allocation in a fairly rich life-cycle model. The importance of housing investment for households is high since housing constitutes an important part of households' portfolios. It is both an investment asset which enables owners to hold home equity, and a durable consumption good from which households derive utility. In order to fully understand the effects of housing investment on portfolio allocation, we developed a fairly comprehensive life-cycle asset allocation model where many important features are taken into account.

The main results of the model can be summarized as follows. First, we show that housing investment has strong crowding out effect on investment in risky assets and this effect is observable throughout the life-cycle. Early in life, households labor income is relatively low and they are willing to be homeowners by paying a downpayment and keep paying annual mortgage payments. So they are liquidity constraint and put some of their labor income mostly in the riskless form due to precautionary motives. Their risky asset share increases gradually over the working life. Even after accumulating enough wealth to pay the fixed cost and begin investing in risky assets, the portfolio's share of risky assets is still at lower levels than predicted by the model with NHI. Hence, owner-occupied housing is a substitute for investment in risky assets. Second, the unconditional and conditional risky shares over the life-cycle under the EZ preferences match the empirical data better than the same shares under the CRRA utility form while we observe similar patterns for the participation rate and homeownership rate for these two cases. Third, the effect of the presence of housing investment on households portfolio allocation is larger than the effect of disentangling the RRA from the intertemporal elasticity of substitution through using EZ recursive preferences.

We further find that, in the absence of house price risk, households invest more in housing and the homeownership rate goes up. Similarly, the share of investment in the risky assets goes down because households enjoy higher and riskless return on housing investment on top of the utility that they generate from housing. We further find that the size of the fixed entry cost on risky asset investment has limited impact on portfolio allocation.

Some extensions of the model for future research include focusing more on the real estate side of the model by analyzing the size and the effects of the liquidation cost, introducing an exogenous mandatory moving and selling of houses (job relocation, old age, health issues, etc.), and allowing households to default on their mortgage. Furthermore, estimating the size of the fixed entry cost and introducing more realistic house price dynamics are left for future research.

APPENDIX: SOLUTION OF THE MODEL

We begin by discretizing the state space and variables over which the choices are made. As each period is 1 year, age (t) is a discrete state variable taking 81 values and mortgage debt (RT) takes 25 values. We discretize liquid wealth using (LW_t) using 17 grid points and use denser grids for low levels of liquid wealth to capture the higher curvature of the value function at this region and we use 11 grid points to discretize the house size (H_t). The random variables (i.e., shocks to labor income process, shocks to return on risky asset, and shocks to return on housing investment) take two possible values. The other state variables, risky asset market participation status ($IFIX_t$) and homeownership status (O_t) also take two possible values. All variables are normalized by the mean of permanent component of labor income ($\bar{F}_t$).[12]

At each point in the state space, the risky asset participation decision is done by comparing the value function conditional on having paid the fixed cost with the value function conditional on having not yet paid the fixed cost. Similarly, the homeownership decision (e.g., house buying or selling decision) is done by comparing the value function conditional on being a renter with the value function conditional on being a homeowner. In both comparisons, adjustments for the payment of the fixed cost of risky asset participation and costs accrued from buying/selling a house (e.g., downpayment, annual mortgage payment, liquidation cost, etc.) are taken into account, respectively.

$$\begin{aligned} v_t(lw_t, h_{t-1}, rm_t, ifix_t, o_t) = \max\nolimits_{0,1}\{ & v_t(lw_t, h_{t-1}, rm_t, ifix_t = 0, o_t = 0), \\ & v_t(lw_t, h_{t-1}, rm_t, ifix_t = 1, o_t = 0), \\ & v_t(lw_t, h_{t-1}, rm_t, ifix_t = 0, o_t = 1), \\ & v_t(lw_t, h_{t-1}, rm_t, ifix_t = 1, o_t = 1)\}, \end{aligned} \tag{A1}$$

where $ifix_t = 1$ and $ifix_t = 0$ denote whether the one-time fixed participation cost is paid or not paid, and $o_t = 1$ and $o_t = 0$ denote whether the household chooses to become homeowner or renters at time t, respectively.

12. The model has a state space dimension of 24,235,200.

We use backward induction to solve the problem because this is finite-time problem. In period $T + 1$, the policy functions are determined by the bequest motive. The value function in this period coincides with the utility function, which is the bequest function. In every period prior to $T + 1$, we obtain the utility function for different combinations of housing, consumption, and other state and choice variables. Then the value function for a typical time t is equal to the utility function of that period plus the discounted expected continuation value ($E_t[V_{t+1}]$). If the continuation value does not lie on the state space grid, we compute the value function using cubic spline interpolation. This backward induction process is iterated from age T to 1.

$$\begin{aligned} v_t(lw_t, h_{t-1}, rm_t, ifix_t, o_t) = \max\nolimits_{c_t, h_t, s_t, ms_t, hr_t, fix_t} \Bigg\{ & u(c_t, h_t)^{\frac{1-\gamma}{\theta}} \\ & + \beta \Big(\mathrm{E}_t \Big[q_{t+1} v_t(lw_{t+1}, h_t, rm_{t+1}, ifix_{t+1}, o_{t+1}) \\ & + (1 - q_{t+1}) w_{t+1}^{1-\gamma} \Big]^{\frac{1}{\theta}} \Bigg\}^{\frac{\theta}{1-\gamma}}. \end{aligned} \tag{A2}$$

Once we compute the value function of all the alternatives, we choose the one that maximizes the value function over all choice variables. The optimum policy rules for consumption, housing, and investment in financial assets correspond to ones that maximize the value function.

LITERATURE CITED

Alan, Sule. (2006) "Entry Costs and Stock Market Participation over the Life Cycle." *Review of Economic Dynamics*, 9, 588–611.

Bucks, Brian K., Arthur B. Kennickell, Tracy L. Mach, and Kevin B. Moore. (2009) "Changes in U.S. Family Finances from 2004 to 2007: Evidence from the Survey of Consumer Finances." *Federal Reserve Board Bulletin*, 95, A1–55.

Campbell, John Y., Joao F. Cocco, Francisco J. Gomes, and Pascal J. Maenhout. (2001) "Investing Retirement Wealth: A Life-Cycle Model." In *Risk Aspects of Investment-Based Social Security Reforms*, edited by John Y. Campbell and Martin Feldstein, pp. 439–82. Chicago: University of Chicago Press.

Cocco, Joao F. (2005) "Portfolio Choice in the Presence of Housing." *Review of Financial Studies*, 18, 535–67.

Cocco, Joao F., Francisco J. Gomes, and Pascal J. Maenhout. (2005) "Consumption and Portfolio Choice over the Life Cycle." *Review of Financial Studies*, 18, 491–533.

Epstein, Larry G., and Stanley E. Zin. (1989) "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns: A Theoretical Framework." *Econometrica*, 57, 937–69.

Epstein, Larry G., and Stanley E. Zin. (1991) "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns: An Empirical Analysis." *Journal of Political Economy*, 99, 263–86.

Fernández-Villaverde, Jesús, and Dirk Krueger. (2011) "Consumption and Saving over the Life Cycle: How Important are Consumer Durables." *Macroeconomic Dynamics*, 15, 725–70.

Flavin, Marjorie, and Shinobu Nakagawa. (2008) "A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence." *American Economic Review*, 98, 474–95.

Flavin, Marjorie, and Takashi Yamashita. (2002) "Owner-Occupied Housing and the Composition of the Household Portfolio." *American Economic Review*, 92, 345–62.

Gomes, Francisco, and Alexander Michaelides. (2005) "Optimal Life Cycle Asset Allocation: Understanding the Empirical Evidence." *Journal of Finance*, 60, 869–904.

Gomes, Francisco, and Alexander Michaelides. (2008) "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents." *Review of Financial Studies*, 21, 415–48.

Gourinchas, Pierre-Olivier, and Jonathan A. Parker. (2002) "Consumption over the Life Cycle." *Econometrica*, 70, 47–89.

Greene, William H. (2008) *Econometric Analysis*, 6th ed. Upper Saddle River, NJ: Prentice Hall.

Grossman, Sanford J., and Guy Laroque. (1990) "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods." *Econometrica*, 58, 25–51.

Guo, Hui. (2004) "Limited Stock Market Particpation and Asset Pricing in a Dynamic Economy." *Journal of Financial and Quantitative Analysis*, 39, 495–516.

Guvenen, Fatih. (2006) "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective." *Journal of Monetary Economics*, 53, 1451–72.

Guvenen, Fatih. (2009a) "An Empirical Investigation of Labor Income Process." *Review of Economic Dynamics*, 12, 58–79.

Guvenen, Fatih. (2009b) "A Parsimonious Macroeconomic Model for Asset Pricing." *Econometrica*, 77, 1711–40.

Haliassos, Michael, and Alexander Michaelides. (2003) "Portfolio Choice and Liquidity Constraints." *International Economic Review*, 44, 143–77.

Hall, Alastair R., Atsushi Inoue, James M. Nason, and Barbara Rossi. (2012) "Information Criteria for Impulse Response Function Matching Estimation of DSGE Models." *Journal of Econometrics*, 170, 499–518.

Hall, Robert. (1988) "Intertemporal Substitution in Consumption." *Journal of Political Economy*, 96, 339–57.

Hansen, Lars Peter, and Kenneth J. Singleton. (1982) "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models." *Econometrica*, 50, 1269–86.

Hu, Xiaoqing. (2005) "Portfolio Choice for Homeowners." *Journal of Urban Economics*, 58, 114–36.

Inoue, Atsushi, and Lutz Kilian. (2013) "Inference on Impulse Response Functions in Structural VAR Models." *Journal of Econometrics*, 177, 1–13.

Khorunzhina, Natalia. (2013) "Structural Estimation of Stock Market Participation Costs." *Journal of Economic Dynamics and Control*, 37, 2928–42.

Laitner, John. (2002) "Wealth, Inequality, and Alturistic Bequests." *American Economic Review*, 92, 270–73.

Leung, Charles. (2004) "Macroeconomics and Housing: A Review of the Literature." *Journal of Housing Economics*, 13, 249–67.

Longstaff, Francis A. (2009) “Portfolio Claustrophobia: Asset Pricing in Market with Illiquid Assets.” *American Economic Review*, 99, 1119–44.

Mehra, Rajnish, and Edward Prescott. (1985) “Equity Premium Puzzle.” *Journal of Monetary Economics*, 15, 145–61.

Merton, Robert C. (1969) “Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case.” *Review of Economics and Statistics*, 51, 247–57.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel. (2007) “Housing, Consumption and Asset Pricing.” *Journal of Financial Economics*, 83, 531–69.

Samuelson, Paul A. (1969) “Lifetime Portfolio Selection by Dynamic Stochastic Programming.” *Review of Economics and Statistics*, 51, 239–46.

Vestman, Roine. (2013) “Limited Stock Market Participation among Renters and Home Owners.” 2013 Meeting Papers 549, Society for Economic Dynamics.

Vissing-Jorgensen, Annette. (2002) “Limited Asset Market Participation and the Elasticity of Intertemporal Substitution.” *Journal of Political Economy*, 110, 825–53.

Vissing-Jorgensen, Annette, and Orazio P. Attanasio. (2003) “Stock Market Participation, Intertemporal Substitution and Risk Aversion.” *AER Papers and Proceedings*, 93, 383–91.

Yao, Rui, and Harold H. Zhang. (2005) “Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints.” *Review of Financial Studies*, 18, 197–239.